Resampled Efficiency™ Fallacies

by

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Abstract

This report responds to critiques of resampled efficiency.

With any substantive innovation such as Resampled Efficiency, there are inevitably questions and challenges to current practice and theory that arise.[1] In some cases such issues may raise unwarranted doubts or concerns. This would be unfortunate since investors would miss Resampled Efficiency's practical asset management benefits including enhanced performance and diversification, automatable portfolio construction and monitoring, investment intuitive and marketable portfolios without the need for ad hoc constraints or unrealistic inputs, increased productivity, improved risk and return estimation, and the availability of important new asset management tools.

Research has led to many developments in Resampled Efficiency optimization since it was invented nearly six years ago.[2] Perhaps the most important open issue – the relative power of Resampled Efficiency optimization versus Bayes estimation – was addressed and resolved in Markowitz and Usmen (2003).[3] We now have a much deeper understanding of the power and benefits of the technology and many enhancements are now available.[4]

In this report we focus on common issues and critiques that have been raised. We show that these critiques are misunderstandings or erroneous. When properties of Resampled Efficiency optimization are properly understood, they are consistent with intuitive investment properties and generally highlight serious unnoticed investment limitations of MV efficiency.

Is Resampled Efficiency a heuristic?[5]

Resampled Efficiency is a generalization of MV efficiency. To see this note that in Michaud (1998, Ch. 6) each of the simulated MV efficient frontiers are computed from 216 simulated monthly returns. This is because the original historical return data consisted of eighteen years of monthly returns. By design, each simulated MV efficient frontier reflects the same amount of uncertainty as the original historical return data. However, the number K of simulated returns is a parameter of the resampling process that need not equal the number of periods in historical data. An understanding of the role K plays demonstrates why Resampled Efficiency is a generalization of MV efficiency.

When K is very large, each simulated MV efficient frontier is similar to the MV efficient frontier and Resampled Efficiency is similar to MV efficiency. When K is small, each simulated MV efficient frontier is likely to be very different and Resampled Efficiency is likely to resemble equal or benchmark weightings. The parameter K is a natural way to model investor uncertainty for MV optimization. At the limit, when K is extremely large,

[1] Resampled Efficiency was first described in Michaud (1998, Ch. 6).

[2] Resampled Efficiency optimization was co-invented by Richard Michaud and Robert Michaud, U.S. patent 6,003,018, worldwide patents pending. New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.

[3] They found that Resampled Efficiency beat Bayes estimation in all ten of their tests. Their results are the subject of Michaud and Michaud (2004) February New Frontier Advisors, LLC (NFA) research newsletter. This and other NFA research articles are available at www.newfrontieradvisors.com/publications.

[4] Introductory and advanced issues and new research results and developments can be found at www.newfrontieradvisors.com/publications.

[5] This issue was first raised in Scherer (2002).

the inputs have no estimation error and Resampled Efficiency equals MV efficiency. When K is finite, the inputs have estimation error and Resampled Efficiency reflects MV optimization with uncertain inputs, the case of practical investment interest.

The Forecast Certainty parameter K has a number of interesting asset management applications.[6] To facilitate the user's experience NFA calibrates Forecast Certainty on a 10-point scale: level 1 – high uncertainty, level 10 – high certainty. Resampled Efficiency Forecast Certainty generalizes MV efficiency by allowing the investor to control estimation uncertainty in the optimization process. Resampled Efficiency simply reflects the appropriate MV efficient portfolio conditional on investor uncertainty.

It may also be useful to note that many investment practices could be criticized as heuristics. For example, the widespread practice of imposing portfolio constraints is often a restriction on portfolio optimality without theoretical justification.

Are Resampled Efficiency portfolios unstable?
MV efficient frontiers typically include a relatively small subset of the assets in the investment universe. As a result simulated MV efficient frontier portfolios have many assets with zero weight and the distribution of asset weights is highly right-skewed. This gave rise to the concern that Resampled Efficient Frontier™ portfolios computed from this skewed distribution may have unstable estimation and poor investment properties.[7]

The well-documented superior performance of Resampled Efficiency relative to MV efficient portfolios in simulation studies puts to rest this concern. The results in Markowitz and Usmen (2003) provide additional evidence that Resampled Efficiency portfolios are well estimated.

While the skewness of the asset weight distribution is of no serious concern relative to investment performance, it is when estimating Resampled need-to-trade-probabilities or for rigorously estimating asset weight ranges. Fortunately, these issues have been dealt with and resolved. Michaud and Michaud (2002) introduce a "meta-resampling" technique that eliminates the asset weight skewness problem for portfolio rebalancing and monitoring and asset weight range estimation.[8]

Are Resampled Efficiency portfolios over diversified?[9]
Resampled Efficiency outperforms MV optimized portfolios because they are better diversified. Resampled Efficiency portfolios are optimal with respect to a large number of statistically equivalent optimization inputs instead of a single set. When properly certainty conditioned, Resampled Efficiency has precisely the correct amount of diversification relative to the assumed level of certainty in the inputs.

6 Patent pending.
7 This concern was raised in Scherer (2002).
8 Patent pending.
9 The overdiversification critique was first raised in Scherer (2002).

The "overdiversification" critique comes from the observation that at the limit, when many simulated efficient frontiers are computed and averaged, every asset in the investment universe will have some weight (however small) in the Resampled Efficient Frontier. In contrast, note that MV efficiency often ignores a substantial fraction of the assets in the optimization universe. However, this property is a feature of Resampled Efficiency not a limitation. In practice analysts devote considerable effort to find attractive investments for the optimization process. The purpose of the optimization is to find optimal weightings of attractive investments and not to ignore them.

Resampled Efficiency does, however, pose a practical problem: how to avoid assets with insignificant or uninvestable asset weights? But this is a familiar problem and various methods are also used in traditional MV efficiency to transform optimal into investable portfolios relative to threshold, increment, and number-of-securities constraints. There are many "investability" algorithms in current commercial use. NFA investability optimization has the unique properties that it begins with Resampled Efficient Frontier optimal portfolios, uses performance similarity as the criterion for optimality, and is compute-efficient for many investment problems of practical investment interest.[10]

Does Resampled Efficiency overweight low-return-risky investments?[11]

Low-return-high-risk assets may have an important role and result in large weightings in many portfolio optimizations. One reason may be that correlation characteristics lead to significant risk reductions. Another reason may be that the asset is a significant part of the benchmark and important in benchmark-relative risk estimation.

However, Resampled Efficiency treats a low-return-high-risk asset differently from MV efficiency. Suppose a very-high-risk-low-return asset relative to others in the investment universe. For example, suppose that the Canada asset in the Michaud (1998) data is assigned a 0.5% estimated return and 10,000% standard deviation.[12] Whatever the Forecast Certainty level, the maximum estimated risk Resampled Efficient Frontier portfolio will have roughly a 50% weighting. This is because very high estimated risk leads to resampled average returns that are equally either way above or way below the estimated returns of other assets.

Does this result imply a serious flaw in Resampled Efficiency? Surely a 50% weighting for a very-low-return-very-high-risk asset is absurd as a constituent of an optimal portfolio. Is this an important example of an unintuitive investment characteristic of Resampled Efficiency? No. The reason, however, leads to some important and interesting investment characteristics of Resampled Efficiency.

As Michaud (2003 fn. 30) notes, the Resampled Efficient Frontier may not be monotone increasing in estimated return as a function of risk. The Resampled Efficient frontier may, in some circumstances, have a maximum estimated return at a "critical" level of risk

[10] Michaud and Michaud (2003b) provide further discussion of the methodology.
[11] The low-return-large-risk asset critique in the context of Resampled Efficiency was first raised in Scherer (2002).
[12] Michaud (1998) data and the Canadian asset was used in Scherer (2002)'s discussion.

beyond which the frontier curves downward and estimated return diminishes. This effect typically occurs when a relatively very-low-return-very-risky asset is included in the optimization universe. Intuitively, Resampled Efficiency teaches that investors should not invest in portfolios beyond the critical risk point.

Michaud and Michaud (2003c) tested the out-of-sample performance of some Resampled Efficient Frontiers that had a critical point. Their simulation studies indicated that out-of-sample investment performance followed the characteristics of the Resampled Efficient Frontier quite well. In specific, Resampled Efficient Frontier portfolios beyond the critical point were found to have increasingly poor average performance relative to portfolios prior to the critical point. Portfolios beyond the Resampled Efficiency critical point are not out-of-sample efficient and should be avoided.[13]

In the very-low-return-very-high-risk Canadian asset example, the Resampled Efficient Frontier allocations to the "Canada" asset for the critical point portfolio are never greater than 2% for any Forecast Certainty level. Resampled Efficiency provides an investment intuitive portfolio even in the investment absurd case Scherer (2002) posits. Other examples we tested were similarly reasonable and intuitively appealing.

It is worth noting that unattractive investments are typically excluded from an optimization a priori. Asset allocation studies rarely, if ever, include low-investment-grade assets such as lottery tickets, avant-garde art, and postage stamps. Such considerations highlight the fact that portfolio optimization is an investment process that requires investment professionals, not mathematical automatons, to reliably add value.

Is Resampled Efficiency useful for Long-Short Investing?[14]

The simple operative principle is that Resampled Efficiency is a generalization of MV efficiency that can be used for any valid application of MV efficiency. Because long-short investing often includes leverage and the assumption of more than normal active risk, it is arguably more important to use Resampled Efficiency in long-short investing than in the long-only case.[15] This is because commercial risk models often have substantial limitations in higher-than-normal active risk MV optimizations and because Resampled Efficiency optimization more realistically estimates risk.

One reason why this misunderstanding arose may be that all the Resampled Efficiency examples in Michaud (1998) are for long-only investing. Another reason is that many portfolio optimization academic studies assume virtually no structure on the portfolio optimization process including no constraints on asset weights.[16] For example, academic studies may allow assets weights of plus or minus a million percent.[17] In this context long-

[13] Interestingly, we also found that the MV efficient frontier portfolios out-of-sample had a similar critical point well approximated by the Resampled Efficient Frontier critical point, though of course not observable on the in-sample MV efficient frontier.

[14] This critique was first raised in Scherer (2002).

[15] Michaud (1993).

[16] Jobson and Korkie (1981) use this assumption and in their subsequent papers on statistical portfolio optimization. A budget or asset sum to one constraint is typically the only constraint. A number of other academic references can be found in Michaud (1998).

[17] For a recent example see Britten-Jones (1999).

short Resampled Efficiency will recommend the same portfolio as MV efficiency. In cases of practical investment interest long-short Resampled Efficiency is different from long-short MV optimization.

Does Resampled Efficiency enhance Performance or Utility?

The claims of superiority in Michaud (1998, Ch. 6) and the results of Markowitz and Usmen (2003) are based on improvements in average investment performance relative to MV efficiency. However, there is a long history of academic studies that focus on optimization performance in the context of expected utility enhancement.[18]

The expected utility criterion has many limitations for portfolio optimization in investment practice.[19] Trivially, Resampled Efficient Frontier portfolios, because they lie below the MV efficient frontier, will always have less "expected utility." More generally, utility estimation is unstable relative to the risk aversion characteristics implied by inexact specification of the form or of the parameter values of a utility function.[20] Additionally, expected utility maximization with Bayesian adjustment for estimation error often leads to choosing less risky efficient portfolios without changing the portfolios on the MV efficient frontier, a solution with little real investment value.[21]

Markowitz and Usmen (2003) use utility functions to identify interesting portfolios on the in-sample efficient frontiers.[22] The performance of the utility-identified portfolios is then measured relative to their out-of-sample average risk and return. The availability of sophisticated Monte Carlo procedures and high-speed computers allows for directly measuring the benefit of alternative portfolio choice in concrete investment terms. Such criteria seem far more appropriate and relevant than the indirect context of utility function studies and their utility function specific results.[23]

Is Resampled Efficiency Consistent with Rational Decision Making?

It is an open question whether Resampled Efficiency is consistent with Von Neumann and Morgenstern (1953) (VM) expected utility axioms.[24] This does not imply that Resampled Efficiency is inconsistent with rational decision making under uncertainty.

We know from mathematical logic that consistent utility axiom systems are necessarily incomplete.[25] It should therefore not be surprising if examples of consistent human

[18] See Bawa, Brown, and Klein (1979). A more recent example is Harvey et al (2003). It should be noted that the Harvey et al utility comparison results are not in the MV efficiency framework and a fortiori are not validly comparable to Resampled Efficiency.

[19] This statement should not be confused with the use of a quadratic function with varying "risk aversion" parameter which is commonly used to compute specific points on the MV efficient frontier.

[20] See for example Rubinstein (1973).

[21] Barry (1971).

[22] Properly identifying comparable portfolios on simulated efficient frontiers raises a number of interesting issues that are beyond the scope of this report.

[23] In their pioneering paper on statistical MV optimization, Jobson and Korkie (1981) use average portfolio performance instead of utility enhancement. In contrast the asset allocation results in Campbell and Viceira (2002) are in-sample and utility function specific.

[24] Note Markowitz and Usmen (2003) comments on this and related issues in their summary.

behavior, such as gains and losses discussed in Kahneman and Tversky (1979), are not consistent with VM axioms. More obviously and relevantly, it is well known that the behavior of professional asset managers is also generally inconsistent with MV efficiency or expected utility maximization.[26]

Unfortunately, the modern notion of rational decision-making is neither well represented nor apparently well understood in much of finance or economics. As Bourbaki (1948) notes, rationality axioms do not define rational thought but follow from our understanding of rational decision-making. The purpose of a set of consistent axioms is to summarize, provide guidance, and help extend our understanding. They in no way limit what can be characterized as rational thought.[27]

As Levy-Markowitz (1979) show, Markowitz Efficiency is a very useful approximation to expected utility maximization. Resampled Efficiency introduces the notion of input uncertainty into the MV expected utility approximation. Forecast certainty is self-evidently a necessary condition for rational use of the Markowitz MV efficient framework. In many cases that we have observed, the behavior of professional asset managers is encouragingly consistent with Resampled Efficiency optimization. If a set of rationality axioms is inconsistent with including forecast certainty in the MV optimization process, then, as in the case of gains and loss behavior, the reasonable hypothesis is that they are incomplete and require revision or replacement.

Conclusion

A number of misunderstandings and errors have arisen in academic and professional publications concerning Resampled Efficiency optimization. These publications have led some to unwarranted concerns and may have limited the benefits available from Resampled Efficiency to many investors and institutional fund participants. In hindsight, these critiques are uninformed or fallacious and resolvable with a deeper understanding of the technology, new research results, and enhancements of the procedures. In fact, proper understanding of Resampled Efficiency sheds much light on many unnoticed unintuitive investment properties of classical MV optimization. Resampled Efficiency is a practical extension of MV efficiency, which has been shown to increase performance and better control risk. Users of Resampled Efficiency should have confidence that their optimized portfolios represent more effective investment management.

[25] Godel (1931).
[26] See discussion in Michaud (1989) and in Michaud (1998).
[27] More discussion is given in the footnotes in Michaud (2003).

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