Comment on "Markowitz versus Michaud: Portfolio Optimization Strategies Reconsidered," Becker, Gürtler and Hibbeln, European Journal of Finance

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We are pleased that Becker, Gürtler and Hibbeln (BGH), authors of "Markowitz versus Michaud: Portfolio Optimization Strategies Reconsidered," are interested in assessing the investment value of Michaud optimization (Michaud 1990, Michaud and Michaud 2008), relative to Markowitz (1952, 1959). As they note, Markowitz and Usmen (MU) (2003) compared Markowitz vs. Michaud efficient frontier optimization in a simulation study. Their results showed that Michaud optimization was superior to Markowitz on average and in every one of the 30 simulation tests performed in spite of the fact that the Markowitz player used a Bayesian estimation procedure to add investment value while the Michaud player did not. BGH also attempt to compare Markowitz to Michaud and, in contrast to MU, finds little evidence of superiority of the Michaud procedure.

However, unlike MU, who carefully followed the simulation framework first used in Jobson and Korkie (1981), and subsequently in Michaud (1998, Ch. 6), and Michaud and Michaud (2008, Ch. 6), BGH used a very different approach. In several important ways, we find that their framework is unsuitable for the issue they claim to test and their conclusions invalid as a consequence. Furthermore, their implementation of the Michaud methods is flawed; as a result, their study is unable to properly discern the preferred optimization method.

Becker et al assess the value of various optimization techniques through a simulation where they estimate the out of sample win rate for each pair of techniques. They do this for a quadratic utility maximizing investor optimizing among 10 stocks in three cases: unconstrained long-short; long-only with a risk free asset; and fully invested long only. While the paper also deals with some issues of interest regarding regularization of the inputs, the main point of the paper is a comparison of Markowitz and Michaud's mean-variance optimizations.

Michaud's optimization technique is a generalization of Markowitz's Mean-Variance (MV) optimization. Consistent with investment practice, Michaud calculates a linear constrained efficient frontier with mean-variance (MV) estimates including estimation error. The resampling method introduced in Michaud optimization can be shown to add investment value in rigorous simulation tests. The Michaud efficient frontier converges to the Markowitz solution as the inputs' error distributions decrease in extent. Although the case of unconstrained MV optimization is not formally defined for Michaud optimization, the definition could be extended to encompass a single portfolio of interest, the maximum Sharpe ratio portfolio. However, in the unconstrained case Jobson and Korkie (1981) have shown convincingly that MV optimization is generally far worse than equal weighting and is not recommendable in practice.

Surprisingly, in spite of referring to Jobson and Korkie (1981), BGH examine the unconstrained MV optimization framework to compare Markowitz and Michaud. But unconstrained MV optimization is not relevant to either Markowitz or Michaud efficiency and their results are totally without merit for comparing the two procedures.

In a second set of tests, BGH consider MV optimization assuming a risk free rate. In this case BGH are using a Capital Market Line framework that is also irrelevant to comparing Markowitz and Michaud. The problem of maximizing utility for an investor with a risk-free asset largely decomposes into choosing the appropriate amount of cash for a given utility function. In all of Markowitz' writings, his framework of choice is the no-risk-free-rate efficient frontier (e.g., Markowitz 2005). In addition, the risk free rate assumption is highly dubious in practice.

BGH also consider a more relevant optimization framework of inequality constrained and no risk free rate MV optimization. However, their framework for the comparing the two optimization procedures uses a specified quadratic utility function and a miscalculation of the Michaud optimal portfolio.

Markowitz has published a number of papers on the relationship of utility maximization to the Markowitz efficient frontier. This is the reason why MU carefully controls the Monte Carlo simulation experiments to determine the right amount of risk in each simulated scenario. Given the BGH framework of only ten equities in the optimization universe with much more volatility than in MU, the fixed quadratic utility function among the more volatile simulations of Michaud will cause a highly variable and noisy portfolio choice for each constituent frontier, as well as having an unreliable relationship as judge to each trial across different simulated truths. For the specific utility function used to score the winner, many simulated frontiers will maximize the utility at the minimum risk portfolio, and many others at the maximum return portfolio. This added noise from a fixed utility optimization is an important reason why Michaud (1998, 2008a) recommends computing an entire efficient frontier without a specific utility function, and allows the investor to choose a portfolio from the frontier (potentially using a utility function at this step). In this scheme, constituent frontier points are associated by rank within a suitable partition of the frontier rather than selected by maximizing a fixed quadratic approximation of utility. Moreover, Markowitz notes (Levy, Markowitz (1979); Kroll, Levy, Markowitz (1984)) that the correct characterization of quadratic utility maximization is that the true utility at each point on the Markowitz (or Michaud) efficient frontier can be approximated by a quadratic function, but that approximation varies point by point. In other words, a fixed quadratic utility function, itself an approximation of the true utility at a specific frontier point, does not apply across the entire frontier or provide a valid comparison among different frontiers optimized from different input information. Consequently, the results for BGH are fallacious and irrelevant from the point of view of ignoring enormous utility noise in their simulations and ignoring proper understanding the role of utility maximization for efficient frontier optimization. We have numerous published (e. g. Michaud 2008a) and unpublished results showing superior out-of-sample performance of the rank-associated implementation of Michaud over the utility-associated version, as well as its superiority over basic Markowitz optimization. A preliminary replication of their experiment with a correct implementation of Michaud showed a total reversal in outcome from those presented by BGH.

We would also like to comment on the scholarship of multiple references in BGH that seemed to minimize the investment value of Michaud optimization. There are published rebuttals of Fletcher and Hillier (Michaud 2003), Harvey et al (Michaud 2008c) and many other papers reviewed in their paper. Also the framework described in Michaud and Michaud (2008) which includes the case of long-short investing simulations is also ignored. We agree that the Scherer (2006) paper is invalid given, as they note, that all the estimation error in the covariance matrix is ignored.

We conclude that the BGH study ignores a contemporary understanding not only of Michaud but also of Markowitz optimization that calls their results into question. As we noted in our discussion of Campbell et al (Michaud 2008c), we consider the conclusions of MU the as-yet definitive study comparing Michaud with Markowitz. The subtleties of designing an experiment properly and implementing the techniques faithfully cannot be ignored, or the results of the experiment will be worthless and misleading, possibly resulting in large investment underperformance.

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