PENSION FUND FINANCIAL PLANNING

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1. Introduction.

A number of recent events have led corporate sponsors of defined benefit pension plans to search for improved financial planning techniques. These events include the trend to expand benefits, depressed equity markets, high rates of inflation and the passage of the Employee Retirement Income Security Act of 1974 (ERISA) which has often resulted in expanded vested benefits, higher pension contributions, and increased fiduciary responsibility. In many cases pension contributions have become a significant burden on corporate financial resources.

A computer-based model for corporate pension fund financial planning, which attempts to integrate actuarial, funding and investment policy, will be described. From a dynamic simulation of corporate and economic events over a planning horizon, projections of the emerging pension liabilities and cash flow requirements are derived under various assumptions. An appropriate asset mix can then be established which is defined in terms of both the financial obligations of the plan and corporate resources, policy, and risk preferences. The first step of the financial planning study is to develop a model which duplicates the process by which the actuary determines pension liabilities. Changes in the workforce and salary levels are simulated over time under a variety of economic and corporate policy conditions, including assumptions about inflation and projected corporate hiring policy. Projected pension liabilities are determined by imposing the actuarial model on

the simulation of the workforce and salary-related events for each year.

Using return assumptions and statistical relationships for pension plan assets based on current expectations and historical data, returns on assets are simulated in order to project the value of pension assets The dual objectives of our analysis are to provide cash flow projections for financial planning and to establish a choice of an asset mix for setting appropriate investment objectives. We will compare the projections of pension liabilities using the pension simulation technique with the standard, constant percent of payroll, projections and show that significant systematic bias can exist in the latter method. We will also document the volatility inherent in the pension liability projections. The results of the simulations which project pension contributions and funding status over the twenty-year planning horizon under a variety of assumptions, will be presented. An analysis of the output will be given, presenting various facets of consideration in the choice of an appropriate asset mix and assumptions for the plan. Our financial planning analysis will focus on pension contribution and cost control as an important part of the investment policy decision. This may appear to contradict the intent of the Pension Reform Act, which

mandates that investment policy shall be for the sole benefit of the plan beneficiaries.[1] However, pension benefits are (essentially) independent of the gains and losses in the pension fund. As long as the plan does not terminate, plan participants continue to receive promised benefits. In the present ERISA environment, basic benefits are guaranteed by the PBGC. Nevertheless, plan termination, if associated with corporate financial difficulty, will probably result in the loss of at least part of the capital value of the, pension plan fringe benefit. Therefore, pension plan beneficiaries are well served by a pension fund investment policy which allows the corporation to continue to fund the pension plan on a long term basis.

Certain basic assumptions are required for a financial planning study to be useful. The essential notions are that of the ongoing concern and that of the fundamental multiperiod character of pension plans. Pension plan liabilities typically mature over twenty- thirty- and forty-year periods. The multiperiod nature of the cash flow projections are useful to a firm that is concerned with meeting future, as well as present, cash flow requirements. Assets are invested in order to meet the

[1] See the Harvard Law Review, Notes, March 1975 for a discussion of this issue.

long term obligations of the plan. Some fundamental changes occur in the return and terminal wealth distribution in a multiperiod framework (see Section 6). As a result, we will show that the effect of various assumptions may be apparent only over extended periods of time.

For the ongoing concern, the economic and social values of funding the pension plan, as opposed to leaving substantial unfunded liabilities with the ever present threat of plan termination can be substantial. There can be considerable tax advantages in funding a pension, plan (Tepper and Affleck, 1974). The social effects of plan termination may be significant as well, especially for past and present employees who may lose benefits.[2] The termination of the plan will likely result in an adverse management-labor relationship. In the Sharpe (1976) analysis, plan termination would result in an increase in the payroll by an amount equivalent to the capital value of the pension plan benefits no longer being funded; therefore, plan termination does not necessarily result in an economic advantage. Finally in the present ERISA environment, plan termination can entail the loss of up to

[2] Under the provision of Title IV of ERISA, the Pension Benefit Guaranty Corporation will ultimately insure all pension liabilities. However, it will be a number of years before insurance is implemented.

thirty percent of corporate assets, making plan termination a potentially traumatic event for the ongoing firm.

Recently, an alternative approach to pension fund financial planning has been advanced; articles on the economic value of plan termination using the "pension put" analysis have appeared (Sharpe, 1976, Treynor, 1976 and (implicitly) Black, 1976). These papers are concerned with the valuation of the plan termination option, using valuation models similar to those considered in Black and Scholes (1973), Merton (1973), Cox and Ross (1976) and Merton (1976).

The single-period nature of a pension plan can be described as a put option sold by the beneficiaries to the corporation (under ERISA, the pension put is sold by the Pension Benefit Guaranty Corporation (PBGC)). At the end of the period, if plan assets are sufficient, plan obligations are paid and the pension plan continues for the following period or is terminated by the firm. If there are insufficient funds, the assets of the pension plan are "put" to the employees as partial payment of promised benefits and the plan is terminated. If the value of the put option is high, if plan termination is a likely event, and the economic value of the plan

termination put option exceeds the value of continuing the pension plan as part of the ongoing concern nature of the firm, then, as Sharpe (1976) points out, funding and investment policy may not matter.[3] Evidently, if single-period considerations are of overriding importance, and the market for pension liabilities is relatively efficient, then a financial planning study of the type described in this report will be of little value. In most cases, the multi-period nature and unique characteristics of the pension plan in the employee benefit package and corporate financial structure would imply that financial planning of the type described in this paper is of direct benefit to the plan sponsor.

Section 2 describes the actuarial methods of the ABC Pension Plan.

The investment policy or asset mix decision has often been addressed in terms of rate of return objectives. Section 3 examines the limitations of this approach, with emphasis on the special characteristics of pension plans.

In Section 4, the nature of the constant percent of payroll cash flow projection method, which is in general use, is described. The assumptions, usefulness and

[3] An exception may exist if the put option is mispriced by the PBGC which would create the possibility of an optimal corporate funding and investment policy.67

possible limitations of the technique are discussed.

In Section 5, the economic basis of the pension simulation technique, with respect to projecting asset returns is briefly reviewed.

In Section 6, the nature of the simulation technique as a "statistical experiment" is considered with the problems inherent in experimental design. A tool which provides a working hypothesis for the simulation results will be discussed.

In Section 7, the methodology of the financial planning study of the ABC pension plan is described in some detail.

In Section 8, an analysis of selected results from the ABC pension plan study is given, including projections of pension liabilities and an evaluation of plan assumptions and asset mix.

In Section 9 we provide a summary of our results.

2. Description of the ABC Pension Plan.

The ABC pension plan is a defined benefit plan, with a salary-based projected benefit cost method, with individual entry age normal as the funding basis and type. As a defined benefit plan, the corporate sponsor acts as an insurer, guaranteeing benefits to plan participants.

Accrued benefits are based on a projection of final five years average salary. Pension liabilities and costs are calculated for each individual in the plan. The "entry age normal" funding type defines the method by which the "normal cost" is established. A normal cost factor is computed for each individual upon entry to the plan. Normal, cost (in this case) is keyed to actual salary at each point in time; normal cost equals salary times the normal cost factor.

The funding requirements in any given year contain four components: 1) the plan's normal cost for the year; 2) an amortization payment to cover the required funding of past and prior service liabilities; 3) a contribution to offset gains and losses due to changes in actuarial assumptions; 4) amortization payments to cover experience gains and losses. In our simulations, pension contributions will consist of normal cost for the year, an amortization of experience gains and losses, and a component consisting of an amortization of unfunded liability which is fixed over the twenty year horizon, beginning in plan year 1976. There are approximately six hundred and fifty participants in the workforce and one hundred eighty vested terminateds and retirees. The actuarial interest rate assumption is eight and one half

percent; the salary growth rate is seven percent.

Other key actuarial concepts (defined for the ABC pension plan) are: benefits, present value of expected benefits, actuarial liability, vested liability and unfunded liability. Benefit payments are the estimated stream of payments the corporation is likely to make to all generations of retirees. The present value of expected benefits is the discounted value of these anticipated benefit streams. Actuarial liability is the value, under the actuarial assumptions, of the assets that would be required for fully funding the promised benefits of the pension plan, at each point in time. For the participants in the workforce ("actives"), the actuarial liability is the present value of expected benefits minus the present value of expected future contributions. For the retirees of the plan, the actuarial liability is simply the value of the lifetime pension annuity that they receive. Vested liability is the present value of vested benefits of the plan/ assuming plan termination, and using current rates for valuing pension liabilities. Unfunded liability is the difference between the actuarial liability and the value of pension assets.

3. Return Objectives and Asset Mix.

Return objectives have often been used to determine the asset mix for a pension fund portfolio. This approach entails selecting asset mixes on the basis of of various return criteria; i.e., maximizing the probability of meeting a target rate, maximizing expected return per unit risk, maximizing expected compound return, etc. They all have in common the characteristic that they isolate the analysis of the investment process from their financial implications. The result of our analysis will be to suggest that the normal course of the return objective-asset mix decision process should be reversed. Using simulation, an asset mix can be chosen which most appropriately matches the emerging financial obligations of the pension plan and the financial resources of the sponsor. Once the appropriate asset mix has been chosen, return objectives can be defined from the simulation results.

The fundamental issue is that a return objective derives its meaning only within the context of its financial implications. A high risk investment policy may imply high return on investments, but the volatility associated with pension contributions may be inappropriate for the corporate sponsor. Alternatively, a low risk investment policy may imply systematic experience losses

which may have a significant cumulative effect on pension contributions. For defined benefit pension plans, the effect of investment policy depends critically on the "actuarial rules of the game": the actuarial asset valuation method, actuarial cost and valuation methods, status of unfunded liabilities, and any experience gains and losses. As a result, it is difficult, at best, to anticipate the effect of an investment policy on pension contributions and plan status.

The problem is compounded by the multiperiod nature of the plan's financial obligations and the necessity of setting investment objectives in a multiperiod framework. The fundamental relationship between risk and long term return is substantially different from the single-period relationship. Increased risk or equity exposure may lead to decreased return on a long term basis (Michaud, 1976). Therefore, single-period return objectives may be of little use in projecting the effect of asset mix on long term return and pension funding.

A return objective stated in terms of compound return may still lead to an inappropriate investment policy for the plan's sponsor. Part of the problem has to do with the unique nature of the pension funding process. It may be assumed that a risk level which maximized expected

compound return over some investment horizon would lead necessarily to a higher expected market value of pension fund assets. Because of the gains and loss analysis, however, another investment policy may lead to higher market asset values due to higher required levels of pension contributions. Alternatively, any return objective however optimized, by not describing the effect of low probability events in financial terms, can lead to seriously inappropriate consequences. For example, it is well known that a portfolio which has a higher probability of achieving a target return than another may also have a larger variance of terminal wealth leading to greater potential for insolvency in the fund and higher funding requirements. In Section 8, we will illustrate some relationships between various return objectives and their financial consequences using our simulation results.[4]

A particular version of the return objective is in current use in pension investment management: asset mix is defined in order to meet the actuarial interest rate. This practice has led many corporate sponsors to transfer significant proportions of their fund assets to fixed

[4] The fundamental theoretical issue involved is that associated with the relevance of maximizing the geometric mean. A significant controversy has developed; see in particular Samuelson (1969), Merton and Samuelson (1973) and Hakansson (1971). The simulation results in Section 8 will be of use in evaluating both the strengths and weaknesses of *a* geometric mean criterion.

income securities and insured funds with yields at or above the interest rate of the plan. The perception is that, ignoring default risk, if portfolio return can be guaranteed to meet or exceed the interest rate, then the growth of assets will lead to systematic gains and, by implication, reduced pension costs.

An important source of error in this procedure is the manner in which the actuarial interest rate has been determined. The actuarial interest rate is often a non-economic risk-adjusted rate. Further, it is standard actuarial practice to offset the effect of one actuarial assumption or approximation against another in such a way that only in totality do the actuarial assumptions and cost and valuation methods make "sense." Frequently a low interest rate is associated with a low salary rate, since their effects when properly balanced, offset each other. Obviously, in such a case, assets that grow at the actuarial rate will not grow sufficiently to compensate for the low salary rate assumption, causing systematic experience losses.

A return objective is additionally inappropriate since it fails to integrate the volatility of the assets with the pension liabilities. An asset mix should be defined which is most appropriate to plan objectives within the

context of the volatility of the net financial obligations of the plan. The inflation rate, for example, which is an important factor affecting the volatility of pension liabilities of salary related plans, can produce significant experience gains and losses. An asset mix consisting of fixed income and insured funds exclusively may provide no hedge against inflation. As a result, pension contributions could be subject to significant volatility as a result of changes in inflation.

4. Cash Flow Projections and the Stochastic Character of Pension Liabilities.

Cash flow projections, where normal costs are based on a constant percent of payroll, are often used to estimate future pension plan costs and liabilities. Given assumptions concerning salary and asset growth rates, and values for the pension liabilities from an actuarial valuation at the beginning of the planning horizon, projections are developed from simple arithmetic computations,[5] derived from recurrence relationships based on the actuarial cost and valuation method for the plan. In Table 1, an example is provided of the constant percent of payroll cash flow projection method for the ABC pension plan. Normal cost is defined, under the plan assumptions,

to be a fixed constant percent of salary. Therefore, normal cost grows at the salary rate of seven percent.

The effect of alternative salary and interest rate assumptions for meeting necessary future contributions to the plan can be evaluated. Cash flow projections of this type can be a useful and time saving approximation for integrating actuarial assumptions with corporate policy in certain cases. However, constant percent of payroll projections also have important limitations: a) the projections are (first moment) point estimates which ignore the volatility and distribution of the estimates; b) the assumptions underlying the rationale of the method, including fixed salary and asset growth rates, are generally unrealistic; c) the method assumes unchanging workforce characteristics; when this assumption is inappropriate, systematic and significant deviations from the point estimates occur.

The projections in Table 1 ignore the components of volatility inherent in the actuarial estimation process. Sources of volatility from actuarial expectations include: a) mortality experience; b) turnover experience;

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[5] Benefit payments are based on the individuals in the workforce near retirement.

TABLE I

ABC RETIREMENT PLAN

CASH FLOW PROJECTION OF PLAN COSTS* ($000)

YEAR	ASSETS	ACT LIAB	NORM COST	BEN	CONTRIB	PAYROLL
1975	7903.	10244.	539.	406.	539.	9607
1976	8815.	11276.	577.	428.	777.	10279
1977	9928.	12414.	617.	421.	818.	10999
1978	11185.	13700.	660.	424.	861.	11769
1979	12592.	15139.	707.	455.	907.	12593
1980	14133.	16718.	756.	571.	957.	13474
1981	15741.	18368.	809.	615.	1010.	14417
1982	17497.	20173.	866.	697.	1066.	15427
1983	19378.	22109.	926.	740.	1127.	16507
1984	21437.	24230.	991.	767.	1192.	17662
1985	23707.	26571.	1060.	892.	1261.	18898
1986	26119.	29062.	1135.	880.	1335.	20221
1987	28822.	31854.	1214.	906.	1415.	21637
1988	31822.	34955.	1299.	1159.	1500.	23151
1989	34916.	38161.	1390.	1295.	1590.	24772
1990	38220.	41590.	1487.	1337.	1688.	26506
1991	41885.	45396.	1591.	1398.	1792.	28361
1992	45901.	49589.	1703.	1686.	1903.	30347
1993	50085.	53928.	1822.	1984.	2022.	32471
1994	54464.	58502.	1949.	2144.	2150.	34744
1995	59186.	63441.	2086.	2402.	2286.	37176

*8½% interest rate
7% payroll growth rate

c) asset growth; d) salary growth: e) the workforce age-sex-service distribution. For the estimation process in Table 1 to hold, the salary and asset growth rate must equal the actuarial assumptions; mortality and turnover experience must follow actuarial expectations; and the workforce age-sex-service distribution must remain stationary. If any deviation from the actuarial assumptions occurs, an experience gain or loss will be recorded for the plan which will affect unfunded liability and plan contributions.

More than the volatility and deviations around trend values of the cash flow projection estimates is at issue. When the projections in Table I are compared with the simulation results in Section 8, we will show that the population is maturing in such a way as to cause systematic deviations in the normal cost. Under these conditions, the cash flow projection methods may be seriously inaccurate.

Constant percent of payroll cash flow projection methods can be useful if their limitations are well understood and if the assumptions have been checked and found to be reasonable approximations for a given pension plan. In order to define an appropriate asset mix for the

plan, the point estimates for pension liabilities must be supplemented by alternative methods which simulate asset values and the funding of the pension plan using the cash flow projections.

5. The Economic Foundations of the Pension Simulation Technique.

The capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) (see also Farna, 1968, 1973) together with the empirical findings of the "efficient markets hypothesis"[6] provide a number of results of interest with respect to portfolio return assumptions. Efficient markets and CAPM taken together imply that the primary determinants of portfolio behavior are the statistical characteristics and relationships of the assets of the portfolio. As a result, the most important investment decision is the choice of the level of systematic risk (beta) and diversification for the portfolio. Empirical evidence with respect to the hypothesis suggests the use of return estimates based on historical data.

Figure 1 shows the basic historical and theoretical relationship between systematic risk and single-period

[6] See Fama (1970) and Jensen (1972) for a review of the theory and empirical results.

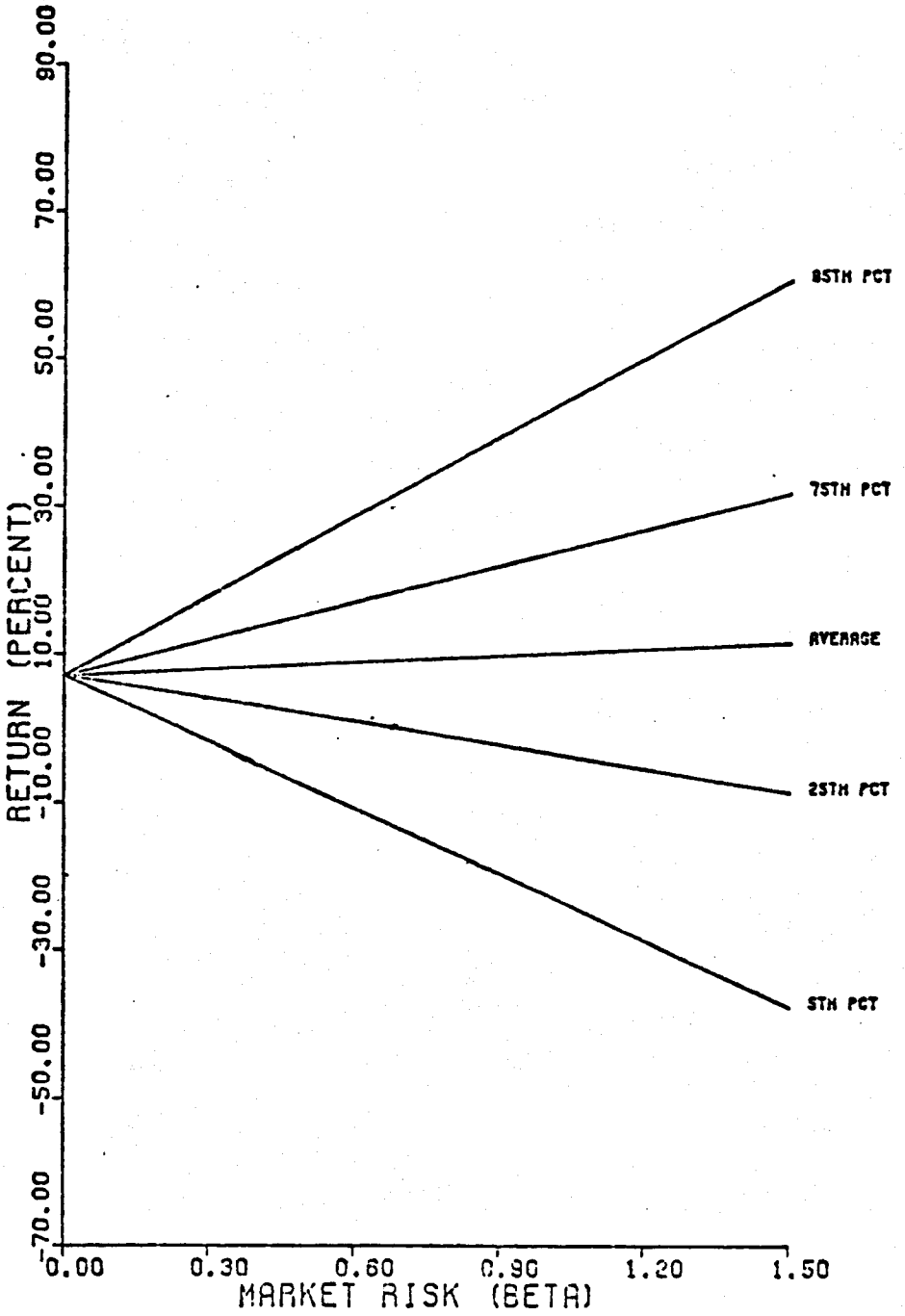
total return. As beta increases, average total return increases. However, there is a concomitant increase in the volatility of the portfolio associated with increasing levels of systematic risk, as illustrated by the percentiles of the return distribution.

The market line model of Sharpe (1963) is a linear return generation process which is consistent with the security market line (SML) of CAPM given in Figure 1. It is this consistency with the theoretical risk-return relationship and the empirical results that provides the economic foundations for the use of the market line model in the portfolio return simulation process.

The market line model is also of use in providing a consistent framework for return assumptions of the assets of the pension plan. It is critically essential to the asset simulation process that assets have consistent risk-return relationships. Otherwise, the simulation results will be predictably skewed toward favoring those assets with a high reward to risk ratio. An asset mix defines a portfolio beta and level of diversification (correlation) via the market line model.

Figure 1

ANNUAL PORTFOLIO RETURN: AVERAGE AND EXTREME VALUES



Under the provisions of ERISA, pension plans may value fixed income securities on an amortized value basis. Assuming that the bond portfolio is passively managed and is on an amortized basis, this implies that any changes in the market value of the bond portfolio can be ignored in terms of experience gains and losses of the plan. As a-result, for the pension plan, the bond portfolio has no volatility with respect to market value. Therefore, excluding default risk, a bond portfolio at amortized value is essentially a riskless asset. Any changes to the value of the portfolio are due to changes in new yield rates for that portion of the bond portfolio which is maturing capital, dividends, or new money allocated to the bond portfolio.

The basic purpose of the bias in ERISA toward passive bond portfolio management is that it provides an opportunity to match fixed rate liabilities with nearly fixed rate assets. Amortized value with a passive bond portfolio strategy is probably the method of choice for valuing and managing the bond portfolio for pension plans. On a historical basis, the passively managed amortized value bond portfolio provides high risk-adjusted return.

6. A Financial Planning Study as a Statistical Experiment.[7]

Monte Carlo simulation is the basic statistical tool associated with the pension simulation technique. Using mathematical-statistical methods, a sequence of (pseudo) random events is generated which has the assumed statistical properties. In particular, Monte Carlo simulation provides a sequence of asset returns with the same statistical properties and relationships which were assumed for the assets of the portfolio.

Monte Carlo simulation, as a statistical technique, is essentially a computer conducted statistical experiment. The principles of the statistical design of experiments are directly applicable. An a priori hypothesis should be formulated as to the effect of various parameters on the statistical experiment. Output should have clear objectives associated with its design.

The multiperiod nature of the pension fund investment planning study introduces some fundamental changes to the single-period return distribution analysis implied by the security market line model illustrated in Figure 1. The appropriate (for most purposes) measure of return over

[7] See Tepper (1974) for an alternative to the Monte Carlo technique.

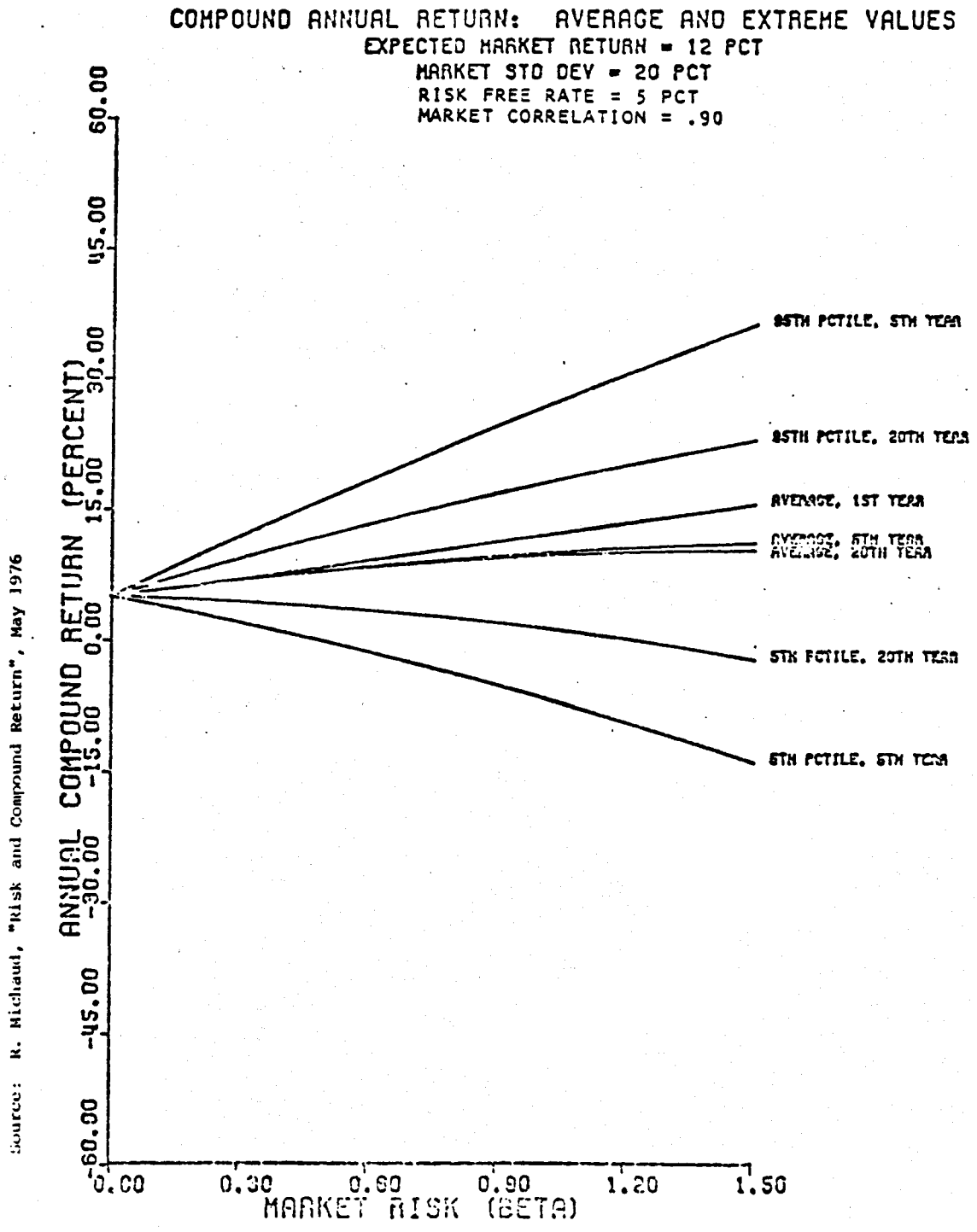
many periods is .the geometric mean or compound return. In Michaud (1976) , an analysis of the relationship between the single-period investment policy variables, beta and diversification, with the distribution of compound return is given. The basic results which have direct relevance for Monte Carlo portfolio (multiperiod) return analysis are: a) compound return (adjusted for cash flows) is asymptotically normally distributed; N-period terminal wealth (adjusted for cash flows) is asymptotically lognormal; b) the mean of (adjusted) compound return is asymptotically directly related to the median of N-period (adjusted) terminal wealth; c) expected compound return is approximately a quadratic function of beta and an increasing function of diversification; d) there exists (generally) a critical value of beta beyond which expected compound return decreases.

In Figure 2, the relationship between risk and compound return is displayed for the indicated values of the parameters. For the parameters given, the critical beta occured at 1.42 for a twenty-year period. In this case, the performance, on a median basis, of an all equity portfolio will be superior to that of an all bond portfolio. The effect of alternative parameters on the risk and compound return relationship is demonstrated in

Figure 3. By changing the variance assumptions on market return a very different situation with respect to the return generation process will result. Under these assumptions, a critical beta of .35 for a twenty-year period results. This would imply that an all-equity asset mix will perform more poorly on a median basis than an all-bond portfolio. These results indicate the critical effect of market assumptions on the simulation process.

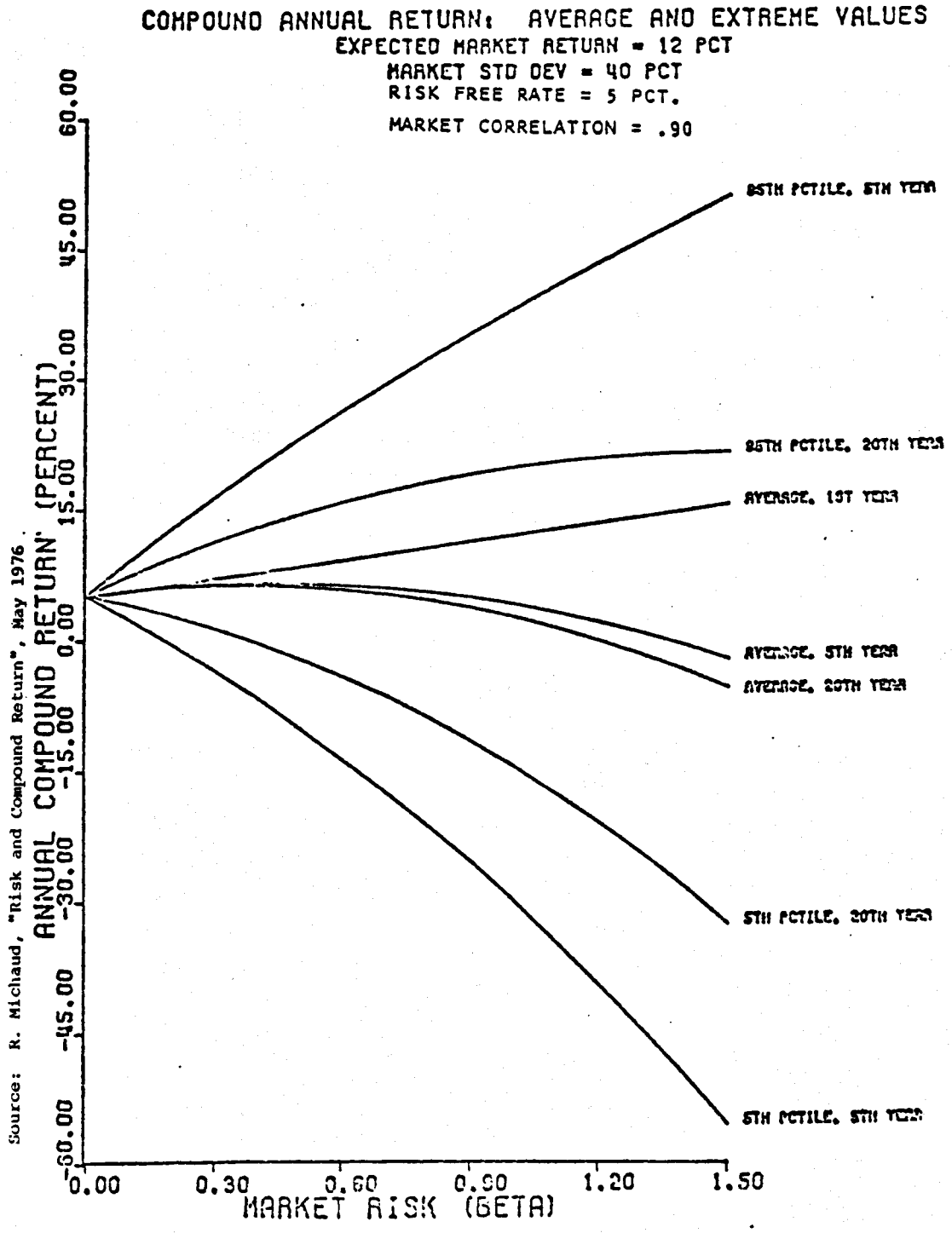
The value of the compound return analysis is that it provides the Monte Carlo simulation technique with an hypothesis concerning the likely results of the simulation. Any deviations from expected results associated with the compound return distribution can be analyzed in terms of the unique factors of a particular pension plan.

The effect of expected compound return on the simulation process can be described intuitively as the amount of "gas" in the return assumptions. Without cash flow effects, and all other things being equal, median terminal wealth should be greatest and median contributions should be least at the asset mix which maximizes expected compound return. However, because of the unique character of defined benefit pension plans,



Source: R. Michaud, "Risk and Compound Return", May 1976

Fig. 2



Source: R. Michaud, "Risk and Compound Return", May 1976.

Fig. 3

such as the gains and loss analysis, maximum median pension asset value may result from an asset mix which requires higher contributions.

In order to ascertain the sensitivity of the results of a simulation to the input parameters, at least three sets of capital market parameters are used. An average set of market parameters, based on historical data, and reasonably consistent with current expectations, serve as a base case. Other inputs which include optimistic and a pessimistic set of assumptions are used to bracket the behavior of the system to the capital market parameter assumptions.

7. ABC Pension Plan Financial Planning Study Agenda.

A financial planning study will generally proceed through four well-defined steps: 1) Establish and validate a computer model of the actuarial valuation process. 2) Set up the assumptions for the pension simulation model; these include projected new entrant age and salary distribution, growth of the workforce, salary growth assumptions, and asset return assumptions. 3) Establish an asset return model and a funding model that simulates the behavior of the funding of the pension plan over the investment horizon, records the actuarial gains

and losses at the end of each period, and determines the pension obligations, funding status and pension contributions. 4) Analyze and evaluate the results of the simulation. We will report on step four in Section 8.

7.1 The Actuarial Valuation Process.

The first step of a pension fund simulation study is to establish a computer model which duplicates the actuarial valuation process. In other words, given the actuarial assumptions, interest rate, salary rate, mortality and turnover, and the workforce age-sex-service distribution, then the computer model generates exactly the same valuation results which would be derived by the plan's actuary. This process entails not only the implementation of standard actuarial formulas for given cost methods, but also must take into account various approximations and alternative discretionary methods which are part of the tools of the actuarial profession. It is a standard actuarial presumption that the valuation methods, assumptions, approximations, and discretionary decisions are interrelated and must be used together. Any changes in assumptions may imply that other parts of the actuarial valuation method have to be changed. As a result, the total actuarial valuation structure must be

carefully constructed so that a valid model of the actuarial valuation process has been established.

Although the results are not reported here, a substantial effort was made to insure that the computer model of the actuarial valuation process was complete and accurate. It was tested under a wide variety of assumptions. Considering the data gathering aspects, consultation with the actuary, and the validation process, implementing the actuarial valuation model can be a significant part of the effort in establishing the pension simulation model.

7.2 The ABC Pension Simulation Assumptions.

ABC anticipates that the total workforce will be stable in size over the foreseeable future. As a result we assumed a constant size workforce for all our simulations. The mortality and turnover tables of the actuarial valuation process are used to simulate the dynamic changes of the workforce over time.

The new entrants' age-sex distribution is an important part of the workforce simulation model. If an individual withdraws as a result of turnover or mortality, a new entrant, under the constant size workforce assumption, replaces that individual. In Table II, the results of a

statistical analysis of the hiring age-sex distribution of the available new entrants data to ABC over the last three years are given. The age groups 21-30 are represented by the new entrant age of twenty-five in the table; the new entrants ages 35-45 are similarly representative. The probabilities in Table II were those used for simulating new entrants.

The actuarial assumptions imply a fixed seven percent salary increase for each individual in the plan. An actuarial valuation, however, keys normal cost and contributions to the actual salary of each individual in each year of the valuation. As a result, stochastic changes in salary increases are reflected in the actuarial liability and costs of the pension plan. The salary rate is traditionally assumed to have four components: inflation, merit, seniority, and productivity. After our discussions with corporate management, it seemed appropriate to consider that the salary rate has two components: inflation and merit. For our simulations we assumed that the effect of inflation over the planning horizon would produce a five percent mean salary growth with a two and a half percent standard deviation; merit would produce a two percent salary growth with three percent standard deviation.

TABLE II

ABC PENSION SIMULATION
AGE-SEX HIRING DISTRIBUTION* ASSUMPTIONS

Entering Age	25	35	45
Males	25%	20%	5%
Females	37%	6.5%	6.5%

* Based on two hundred new entrants data from 1973-75.

The new entrant hiring salary distribution was analyzed similarly to the data in Table II and input to the model. Starting salary was assumed to grow over the investment horizon at the seven percent rate. The social security covered compensation base was assumed to grow at a fixed four and a half percent rate from the $15,300 base in 1975.

Table III displays the average return and volatility of equity investments in a portfolio of market value weighted securities for various historical periods (taken from Scholes, 1975). A number of considerations play a part in our asset return input assumptions. 1) In the light of present economic expectations, inflation will probably persist at a higher level than has been historically realized. 2) Historically/ stocks have an average return-risk premium over Treasury bill rates of at least five to seven percent (Ibbotson and Sinquefield, 1976). 3) Asset return expectations should be consistent with the inflation rate assumption of about five percent, which is part of the actuarial interest and salary rates of the ABC plan. 4) Yields for high quality corporate bonds have historically provided a two percent risk premium over the Treasury bill or inflation rate (Ibbotson and Sinquefield, 1976). As a result, our equity return

assumptions for our average market base case were: twelve percent average total return, twenty-two percent standard deviation and a five percent risk-free rate.

In this paper we consider only a passive strategy for bond portfolio management, with bonds at amortized value. We assumed a seven percent new money coupon rate, with a two percent standard deviation and a negative .3 correlation with the market index. In our simulations, we assumed a twenty percent bond turnover rate. This represents that portion of the bond portfolio which is assumed to be maturing capital and dividend income at the end of any year.

Simulations of asset return, inflation and merit assumed a normal distribution.

7.3 The Operation of the Simulation Model

The fundamental objective is to create a "statistically realistic" environment within which actuarial valuations and pension funding take place over time under a variety of assumptions.

For a given simulation over the planning period, the asset mix, funding policy and market return expectations are fixed. The workforce is aged on an individual basis year by year. For each individual, the model records his

age-sex-service status and whether he continues as an active, becomes a vested terminated, terminates without vesting, retires or dies. If the individual has dropped out of the workforce, a new entrant replaces that individual in accordance with the hiring age-sex distribution assumption.

Given a simulated workforce with simulated salaries and a simulated vested and retired population at each point in time, an actuarial valuation takes place in accordance with the actuarial assumptions and methods. Actuarial liabilities, normal costs, etc., are computed for each individual and are cumulated into totals for the plan. The model then computes the unfunded liability of the plan and pension contributions required under ERISA. Developments in plan funding produce a possible stream of pension contributions and a progression of plan status over time.

The performance of many simulations results in a statistical profile of pension contributions and funding status over the investment horizon. By performing simulations for many fixed (rebalanced) asset mixes, the progression of the funding of the pension plan over the planning period can be compared and an appropriate asset mix chosen.

TABLE III

HISTORICAL ANNUAL EQUITY RETURNS*

PERIOD	AVERAGE RETURN	STANDARD DEVIATION	COMPOUND RETURN
1926-1945	17.8%	41.2%	6.7%
1946-1965	15.1%	19.8%	13.3%
1926-1965	16.5%	32.3%	9.9%
1926-1974	10.5%	22.2%	8.1%

HISTORICAL AVERAGE BOND PORTFOLIO

Annual New Money Rates (Yields)

	AVERAGE RETURN	STANDARD DEVIATION	COMPOUND RETURN
1926-1974	4.5%	1.5%	4.5%

Historical Correlation between New Money Rate or Bond Yields on Long term Bonds and Stock Market Returns 1926-1974: -.29

*Taken from Scholes (1975)

8. ABC Pension Plan Simulation Study: Analysis and Evaluation of Results

8.1 Workforce Simulations.

As a source of volatility in the actuarial valuation process, the ageing and changing nature of the workforce is, in many ways, unique. The dynamic character of the stochastic changes in the workforce at each point in time is dependent on the state of the workforce at the previous point in time.[8] As a result, workforce changes can be a source of systematic bias as well as volatility in the actuarial valuation process.

Simulations of the ABC workforce were performed for one hundred years and are summarized in Table IV. The results show that the ABC workforce will systematically age from a present mean age of thirty-seven to a mean age of forty-four over a twenty-year period. The population appears to stabilize in about twenty years.

In Table V, the entire ABC pension population is simulated, which includes retirees and vested terminateds. Again a significant ageing of the population is evident. We can anticipate a stable population size of about a thousand participants. A stable population appears to

[8] In technical terminology, the ageing of the workforce is a multi-state Markov process.

take longer to achieve; in these simulations at least thirty years is required.

8.2 ABC Projected Pension Liability Distribution.

An analysis of the projected pension liabilities developed from our simulations for the ABC pension plan will be presented. These results will be compared to the constant percent of payroll or roll-forward projections of Table I and to simulations using a seven percent interest rate assumption.

Table VI presents the median results of ABC's projected pension liabilities. Actuarial contributions were computed under the assumption that pension assets grow at the interest rate. Therefore, the projections in Table VI are directly comparable to those in Table I. Table VI shows that there is substantial and systematic bias in the roll-forward projections in this case. Payroll grows at a considerably higher rate: 8.0% on a compound basis over the twenty-year period instead of seven percent under the actuarial assumptions. This result is due to the systematic ageing of the workforce which was exhibited in Table IV and because salary is age-related. Normal cost has a compound growth rate of 9.1% in Table VI over the twenty-year period, versus seven percent in Table I.

TABLE IV

ABC WORKFORCE
SIMULATED AGE DISTRIBUTION*

STATISTICAL SUMMARY

Year	Mean	Median	St. Dev.	Skew
Present	37.4	34	12.8	.48
5	39.4	37	11.9	.45
10	41.2	40	11.5	.26
15	42.6	43	11.4	.08
20	44.4	45	11.5	-.09
30	45.3	47	11.9	-.24
40	44.4	45	12.2	-.10
60	45.5	47	11.7	-.24
80	43.8	45	12.2	-.03
100	44.2	45	12.1	-.10

*Constant Size Workforce

TABLE V

ABC PENSION POPULATION

SIMULATED AGE DISTRIBUTION

STATISTICAL SUMMARY

Year	Mean	Median	St. Dev.	Skew	Total
Present	44.5	41	18.2	.51	830
5	46.2	44	17.3	.58	846
10	48.2	46	17.1	.53	887
15	48.8	46	16.3	.48	891
20	50.4	49	15.8	.42	916
30	52.4	52	15.7	.14	970
40	53.6	54	16.7	.04	1037
60	55.8	55	17.6	.08	1067
80	53.8	54	17.8	.14	1041
100	54.2	54	17.9	.18	1044

There are two important reasons for the normal cost growth rate. Normal cost depends on salary and therefore is age-related. Also, there is a systematic ageing of the entry age normal cost factors due to asymmetries in- the turnover table. The non-uniformity or "lumpiness" in the workforce population data is most vividly illustrated with the benefits projections. Benefits, in any year, depend on the number of individuals in the workforce who are eligible for retirement in that year and their salary-service benefits, upon retirement.

Table VII documents the volatility associated with the ABC pension liability projections. The numbers in the table describe the range about the mean associated with ninety-five percent of the simulations, expressed as a percent of the mean.

With the possible exception of actuarial contributions, projected pension liabilities do not appear to have a very significant degree of volatility, if the ABC pension plan data are representative. In any particular case, if substantial bias due to dynamic changes in the workforce could be ruled out, the roll-forward projections of Table I might be useful.

The results of one hundred simulations of ABC's pension liability distribution for selected actuarial projections are displayed in Figures 4-7. The dashed lines in the figures represent (where appropriate) the roll-forward projections from Table I.

In evaluating the actuarial projections, it is important to recognize that, within a plan year, benefits and pension contributions are cash flows which enter at the beginning or during the year. Therefore, 1975 normal cost, contribution and benefits given in Figures 4, 6 and 7 are fixed cash flows for the 1976 plan year.

Figure 8 documents the effect of a seven percent interest rate assumption on the ABC actuarial contribution distribution, with all other assumptions as before. In particular, median actuarial contributions in Figure 8 jump (approximately) from $800,000 to $1,200,000 in 1976 from $1,700,000 to $2,500,000 in 1985 from the corresponding actuarial contributions under the eight and a half percent interest rate assumption. Evidently, the interest rate assumption has a very significant effect on pension costs.

TABLE VI

ABC RETIREMENT PLAN

PENSION LIABILITY PROJECTIONS ($000)

MEDIAN CASE

YEAR	ACT LIAB	NORM COST	COST %CHG	BEN	VEST LIAB	ACT CONT	TOT SAL	SAL %CHG
1975	10244	539		406	5491	539	9607	
1976	11347	605	12.2	428	5863	773	10506	9.4
1977	12551	674	11.4	421	6260	852	11458	9.1
1978	14126	755	12.0	424	6809	957	12623	10.2
1979	15848	837	10.9	455	7605	1050	13743	8.9
1980	17576	910	9.1	571	8450	1108	14683	6.8
1981	19356	996	9.4	615	9493	1191	15870	8.1
1982	21582	1089	9.3	697	10500	1297	17188	8.3
1983	24047	1196	9.8	740	11873	1452	18704	8.8
1984	27043	1310	9.5	767	13333	1557	20227	8.1
1985	30215	1430	9.2	892	15215	1701	21934	8.4
1986	33554	1552	8.5	880	16910	1860	23652	7.8
1987	37376	1709	10.1	906	19216	2026	25877	9.4
1988	41239	1829	7.0	1159	21789	2158	27413	5.9
1989	45908	1969	7.7	1295	24371	2316	29496	7.6
1990	50075	2125	7.9	1337	27000	2469	31530	6.9
1991	55918	2321	9.2	1398	30710	2731	34459	9.3
1992	62624	2504	7.9	1686	35105	2973	36661	6.4
1993	69146	2659	6.2	1984	39496	3086	39066	6.6
1994	74747	2837	6.7	2144	44085	3294	41754	6.9
1995	83769	3053	7.6	2402	49937	3451	44667	7.0

Interest Rate 8.5%

Wage Growth Rate 7.0%

TABLE VII

VOLATILITY OF ACTUARIAL PROJECTION DISTRIBUTION

RANGE AS A PERCENT OF THE MEAN PROJECTION (±)

(95% Confidence Level)

	Act Liab	Norm Cost	Ben	Vest Liab	Act Cont.*	Tot Sal
3rd year	8%	8%	7%	7%	17%	7%
5th year	11%	9%	12%	10%	22%	9%
10th year	15%	11%	14%	15%	29%	11%
20th year	21%	15%	23%	22%	41%	14%

8½% interest rate
7% salary rate

*Assets are assumed to grow at the fixed interest rate. Experience gains and losses in unfunded liability due to changes in actuarial liability are amortized over fifteen years.

8.3 ABC Pension Plan; Plan Status and Investment Objectives.

In order to project plan status and to define appropriate investment objectives, investment experience must be integrated with pension liabilities and the gain and loss analysis associated with asset return. In our simulation study, asset simulations were segregated from the pension liability simulations. There were a number of reasons for this procedure: a) from a computer cost perspective, the pension liability simulations take substantially more time than the asset simulations, and require far fewer simulations; b) after an analysis of the individual simulations of the pension liabilities over a twenty-year period, it was found that statistical summaries, such as the median, were useful surrogates of the actual simulations; c) the number of assumptions of interest was far greater for asset simulations than for liabilities.

We will analyze the simulation data in detail for a base case set of assumptions with median ABC pension plan liability simulation data. Exhibits from other cases will be selected and compared to this base case.

ABC PENSION PLAN
NORMAL COST
ACTUARIAL PROJECTION DISTRIBUTION

INTEREST RATE .035
SALARY RATE .070

Fig. 4

Source: R. Michaud, "Pension Fund Financial Planning", May 1976

ABC PENSION PLAN
ACTUARIAL LIABILITY
ACTUARIAL PROJECTION DISTRIBUTION

INTEREST RATE .035
SALARY RATE .070

Fig. 5

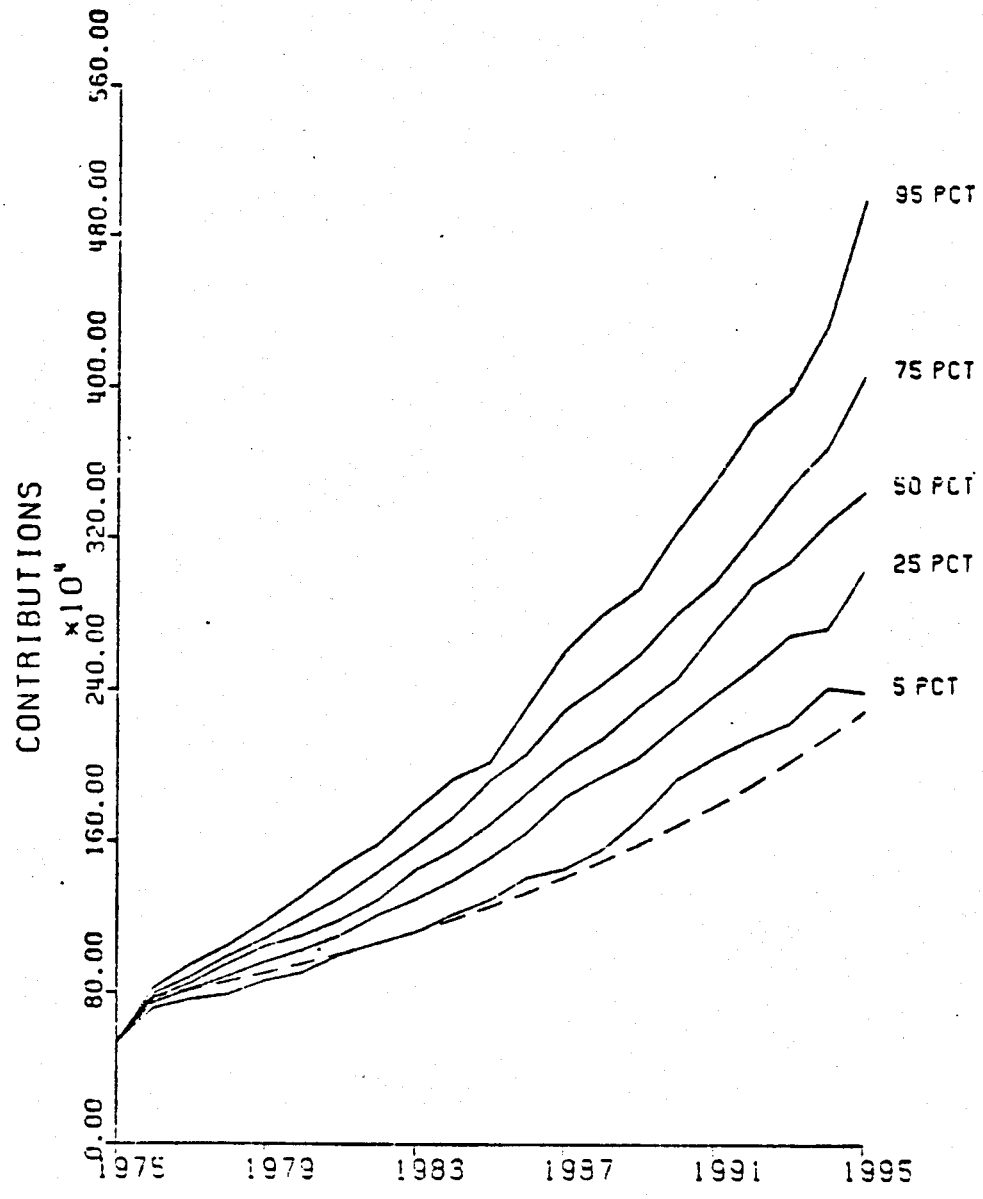
Source: R. Richaud, "Pension Fund Financial Planning", May 1976

ABC PENSION PLAN

CONTRIBUTIONS
ACTUARIAL PROJECTION DISTRIBUTION

INTEREST RATE .085
SALARY RATE .070

Fig. 6



Source: R. Richaud, "Pension Fund Financial Planning", May 1976

C PENSION PLAN
BENEFITS
ACTUARIA ROJECTION DISTRIBUTION

EREST RATE .035
AST RATE .070

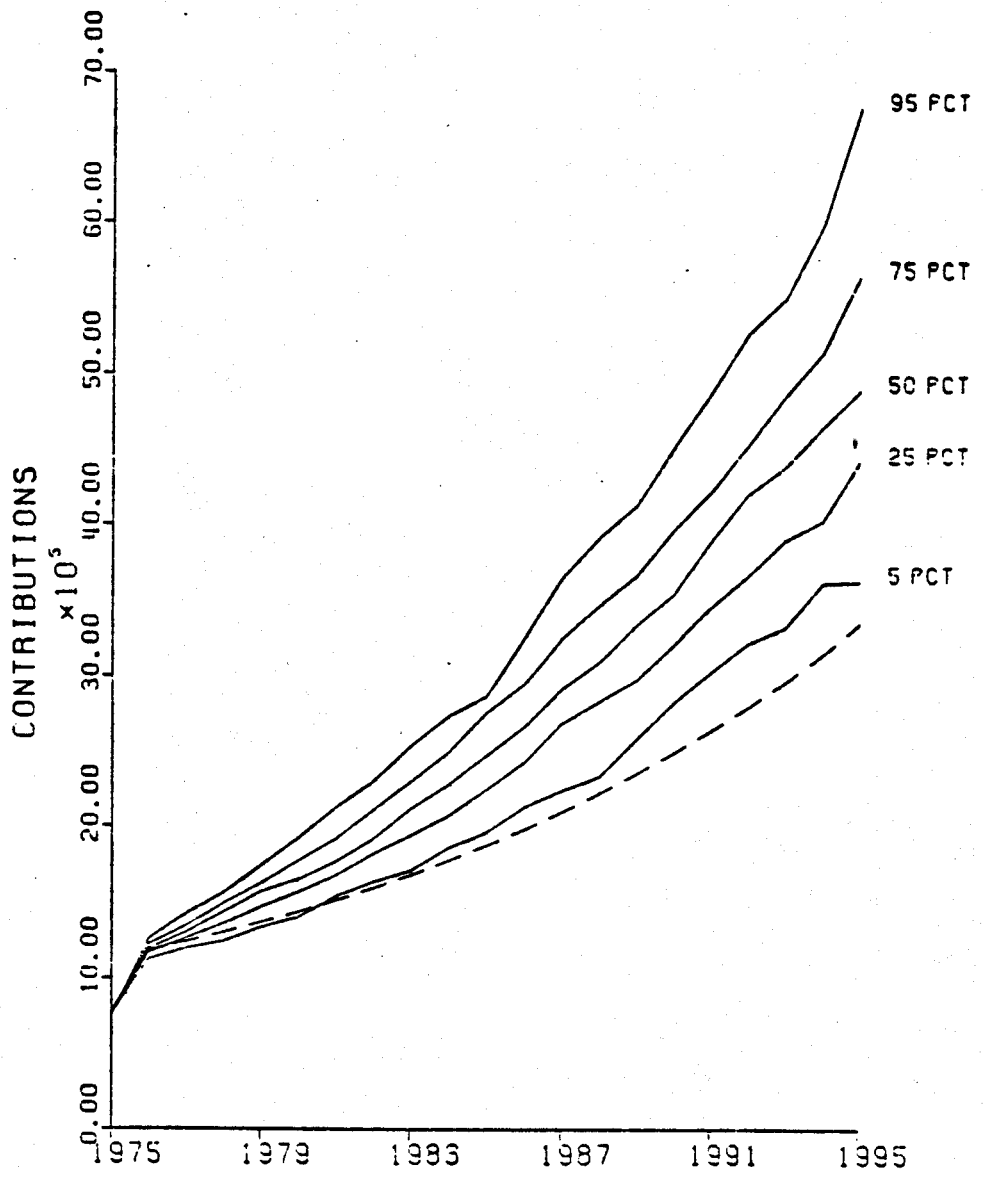
Fig. 7

Source: R. Michaud, "Pension Fund Financial Planning", May 1976

ABC PENSION PLAN
CONTRIBUTIONS
ACTUARIAL PROJECTION DISTRIBUTION

INTEREST RATE .070
SALARY RATE .070

Fig. 8



Source: R. Michaud, "Pension Fund Financial Planning", May 1976

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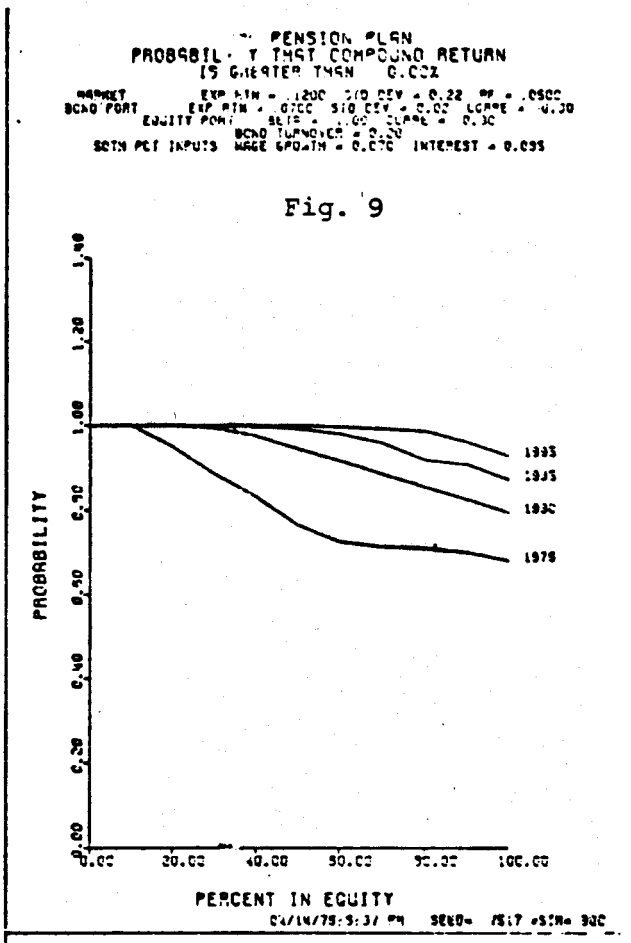
The major body of the simulation data is in the form of graphs which either document the behavior of the pension fund across time for a given percent of assets in equity or compare the effects of various equity positions at a given point in time of the investment horizon. There are numerous ways of examining pension fund behavior. We have used: 1) compound return probability; 2) plan status; 3) pension contributions profile; 4) compound return distribution; 5) contributions as a percent of payroll.

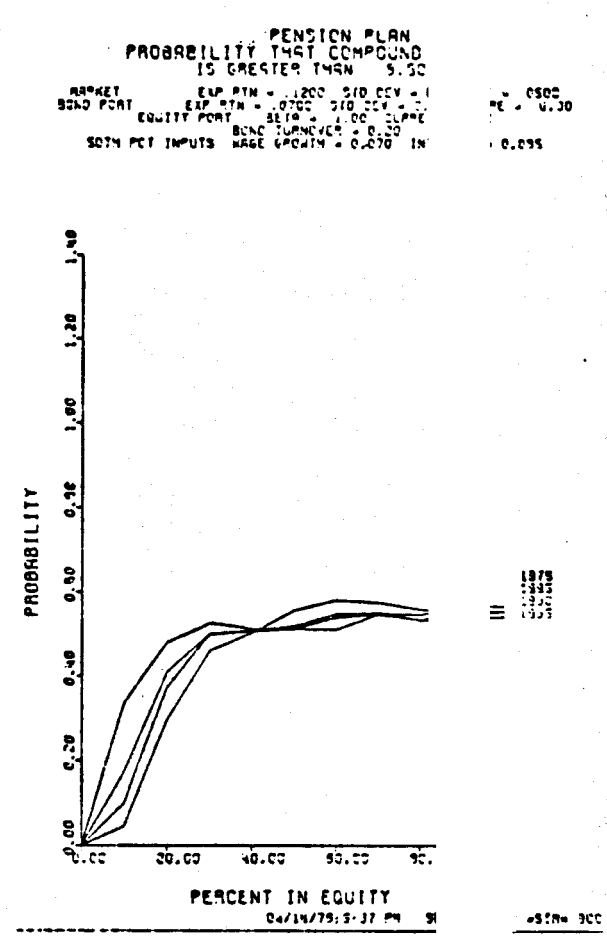
For the simulations which will be presented, the equity portion of the pension portfolio was on a full market valuation basis; ABC's actual pension contribution for plan year 1976 was input.

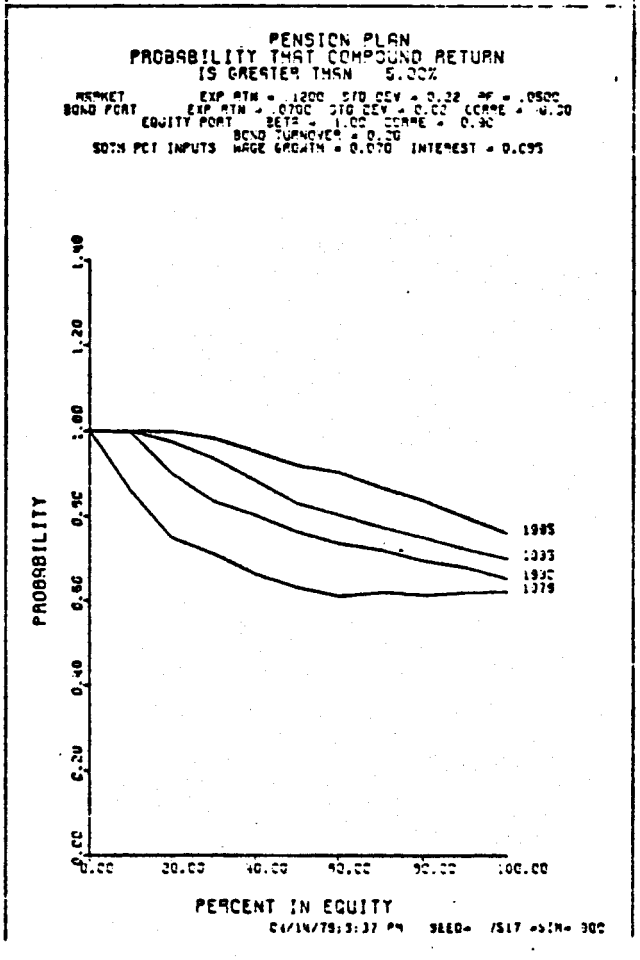
Figure 9 illustrates the effect of various equity investment policies on the inherent risk-return tradeoffs of portfolio behavior. Of particular interest is the probability of achieving the actuarial interest rate. We note that this probability is never greater than sixty percent, even over a twenty-year period. For any given investment horizon, that asset mix which maximizes the probability of achieving the interest rate maximizes the probability of actuarial gains and should be associated

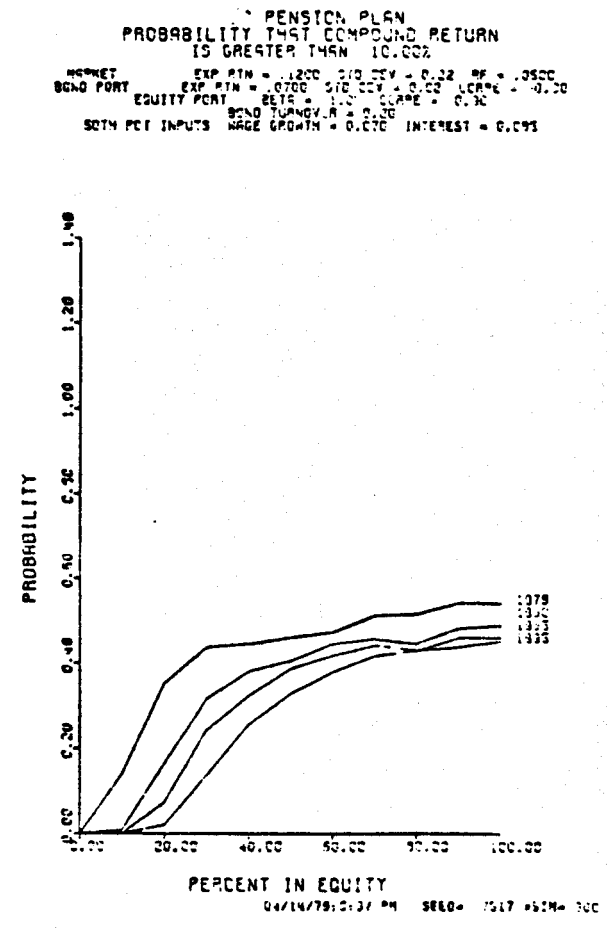
Source: R. Michaud, "Pension Fund Financial Planning", May 1976

Fig. 9









Source: R. Michaud, "Pension Fund Financial Planning", May 1976

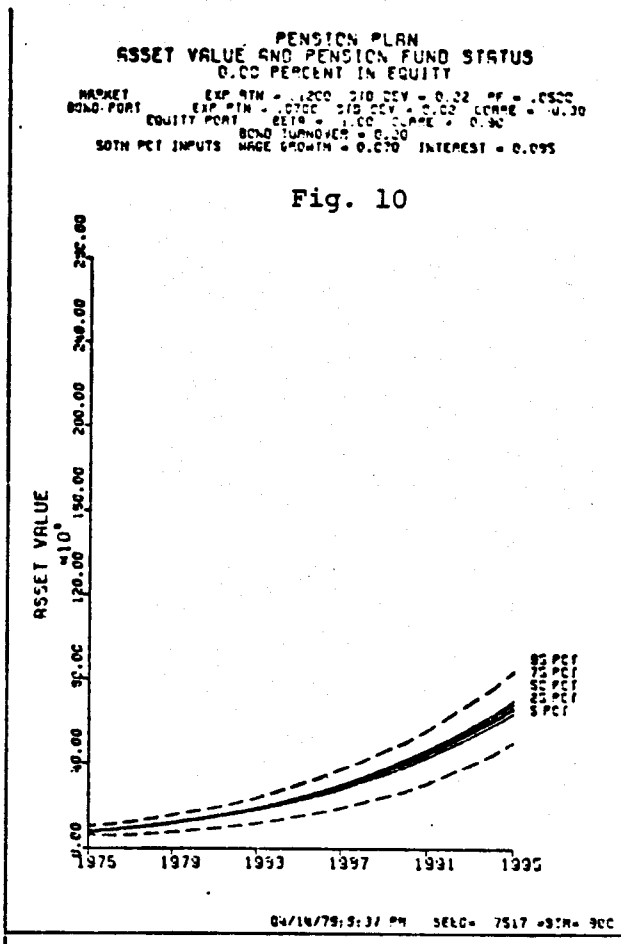
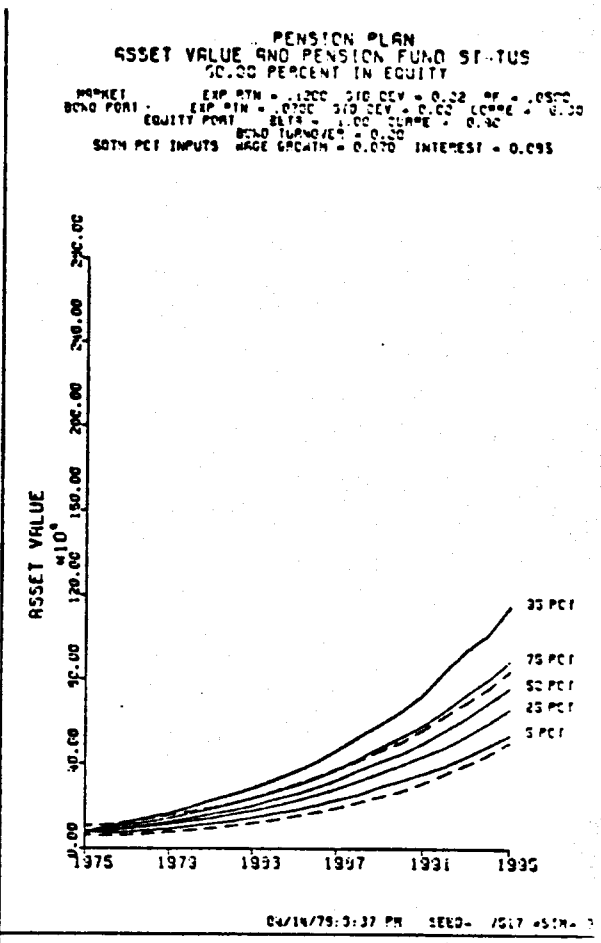
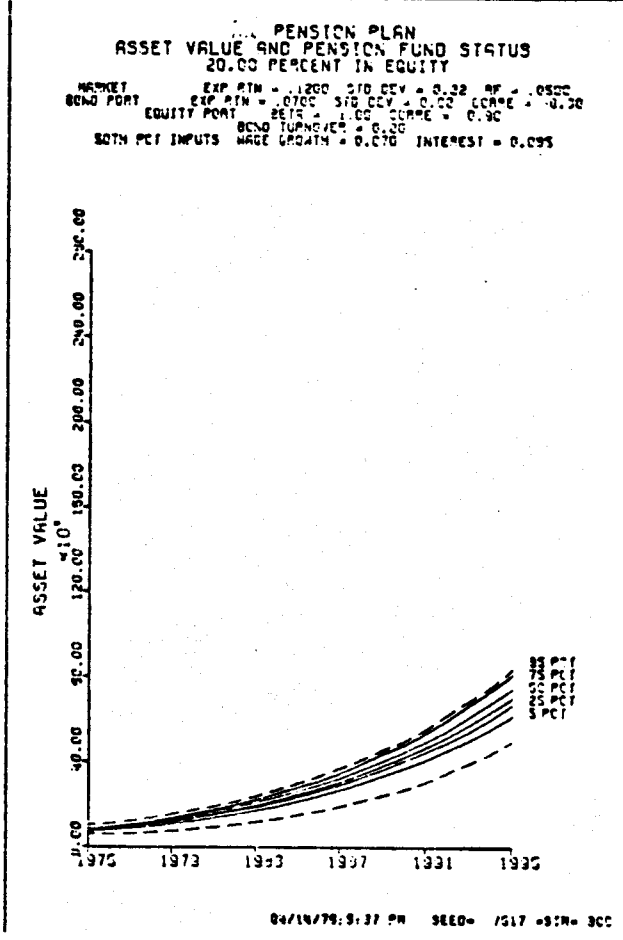
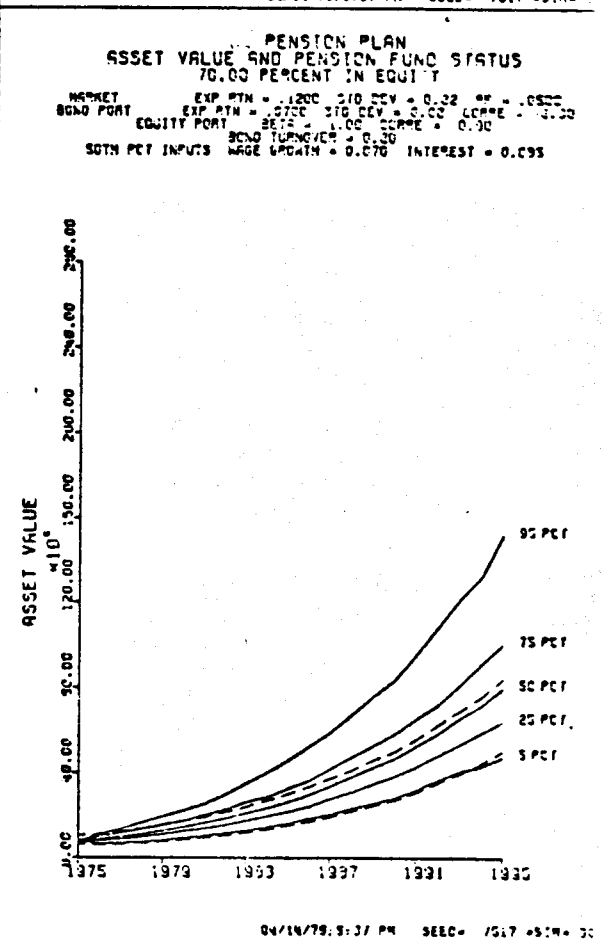


Fig. 10







Source: R. Michaud, "Pension Fund Financial Planning", May 1976

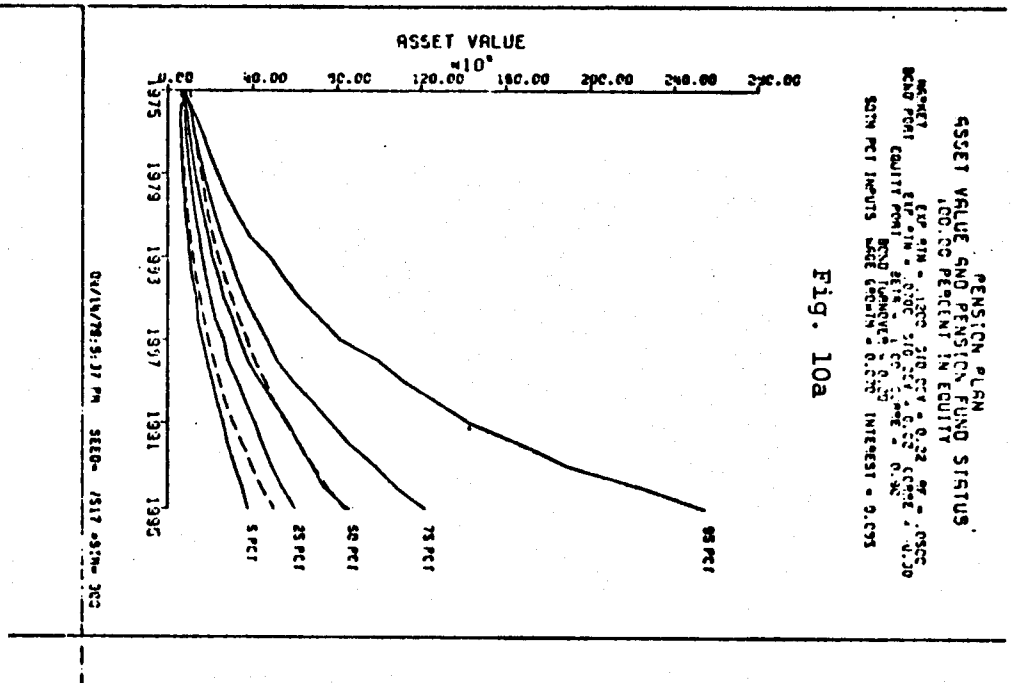


Fig. 10a

with relatively low contributions However, the rate of return objective does not indicate the level of contributions which may result if the return objective is not realized.

Figure 10 illustrates the progression of pension fund status over time for various percents in equity. The top dashed line represents median actuarial liability; the lower dashed line represents median vested liability.

Increased equity effectively increases the probability of full funding, but also increases the probability that asset value will be less than vested liability. The essential lognormal character of terminal wealth is evident, by the upside potential in asset value, especially over long periods of time and for high equity periods.

Figure 11 illustrates the contribution profile over time and compares simulated pension contributions to the median projected actuarial contribution. For small percentages in equity, it is nearly certain that contributions will be above the actuarial level. The effect of increased holdings in equity is to reduce the median level of contributions while raising the possibility of large contributions. The corporate sponsor

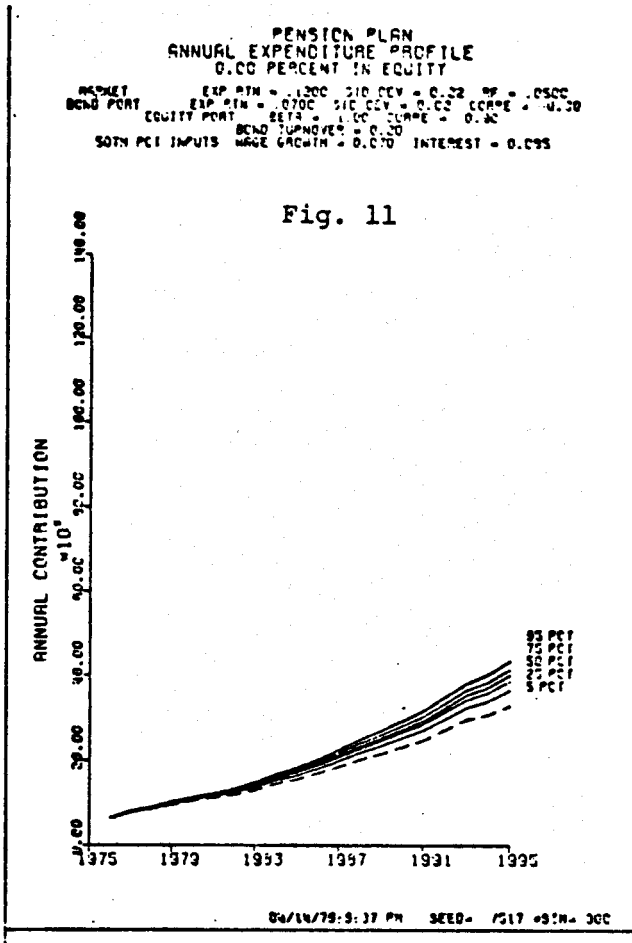
must weigh the value of minimizing median contributions against the possibility of substantially higher contributions.

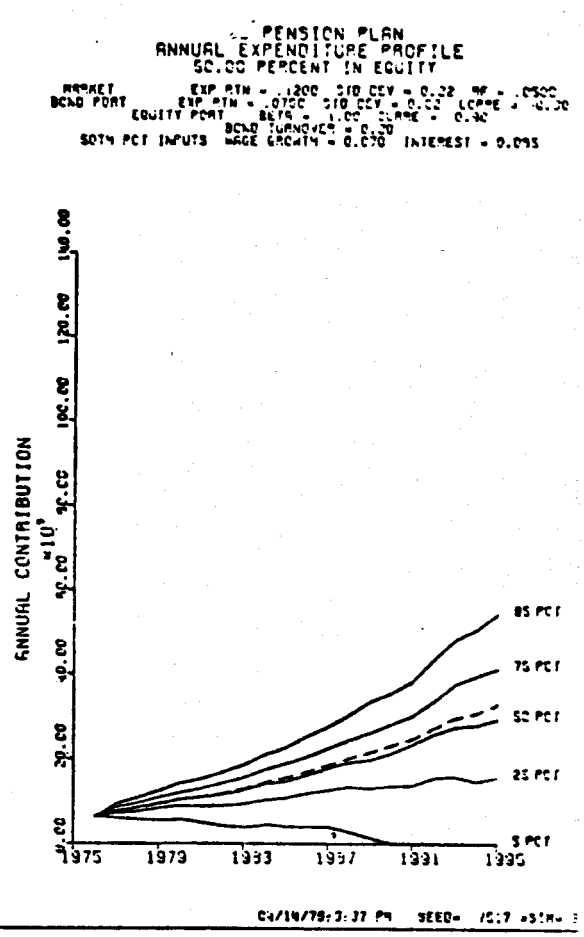
Figure 11 shows the effect of asset mix on portfolio compound return. The asset mix which maximizes median compound return will minimize median contributions. However, the rate of return objective of maximizing the mean or median of the geometric mean does not indicate the level of contribution which may result if the median contribution is not achieved. A highly volatile stream of contributions may result from attempting to maximize the geometric mean, which may not be appropriate for the corporate sponsor. The compound return distribution can be very useful for understanding the underlying statistical character of the simulation process.

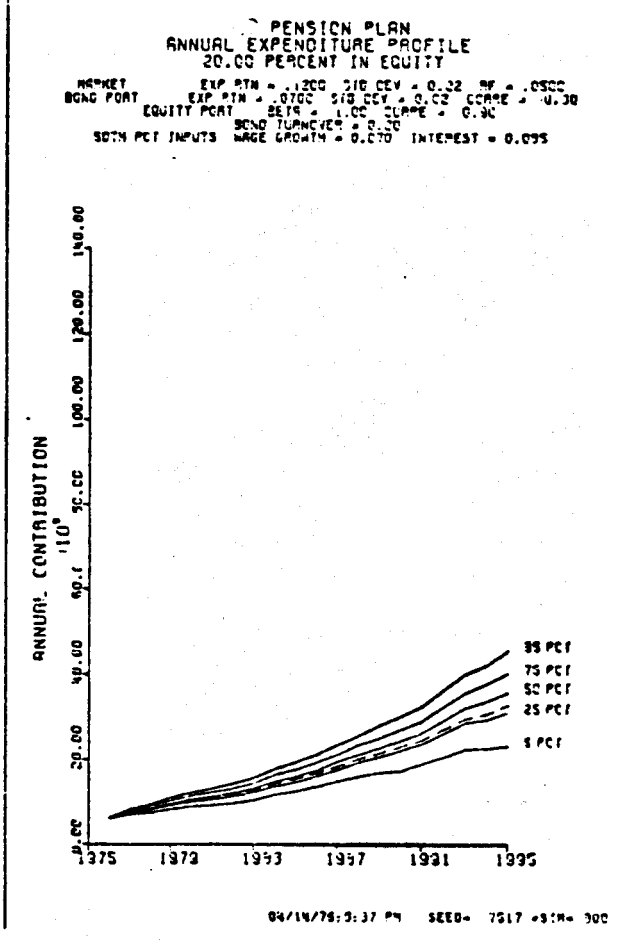
Figure 13 describes the effect of asset mix on contributions as a percent of payroll. If financial planning can be assumed to be a cost control process, and if payroll is a reasonable measure of corporate size, then contributions indexed to payroll can be a useful measure of the impact of pension contributions on corporate financial resources. Although a fairly wide range exists, on a median basis, contributions as a percent of payroll is relatively constant over time.

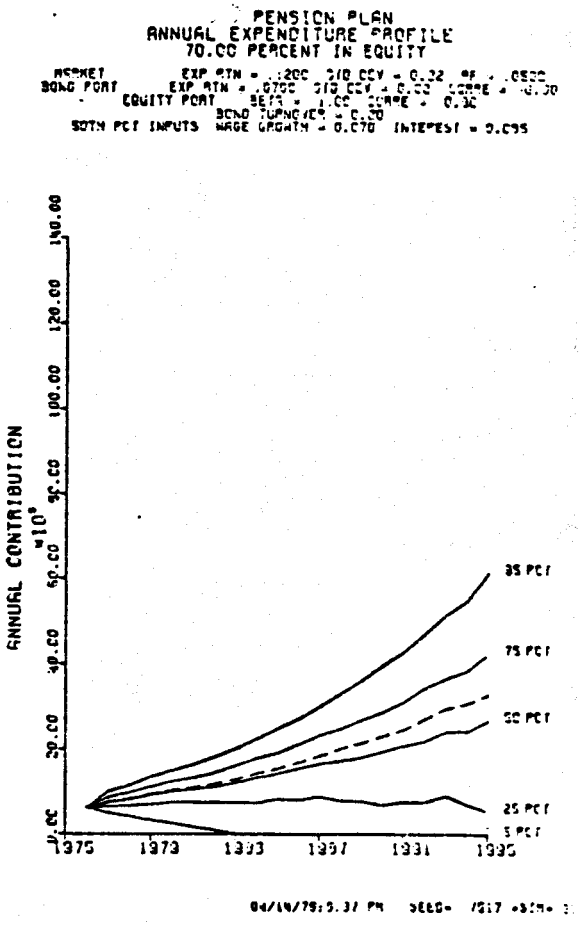
Source: R. Michaud, "Pension Fund Financial Planning", May 1976

Fig. 11









Source: R. Michaud, "Pension Fund Financial Planning", May 1976

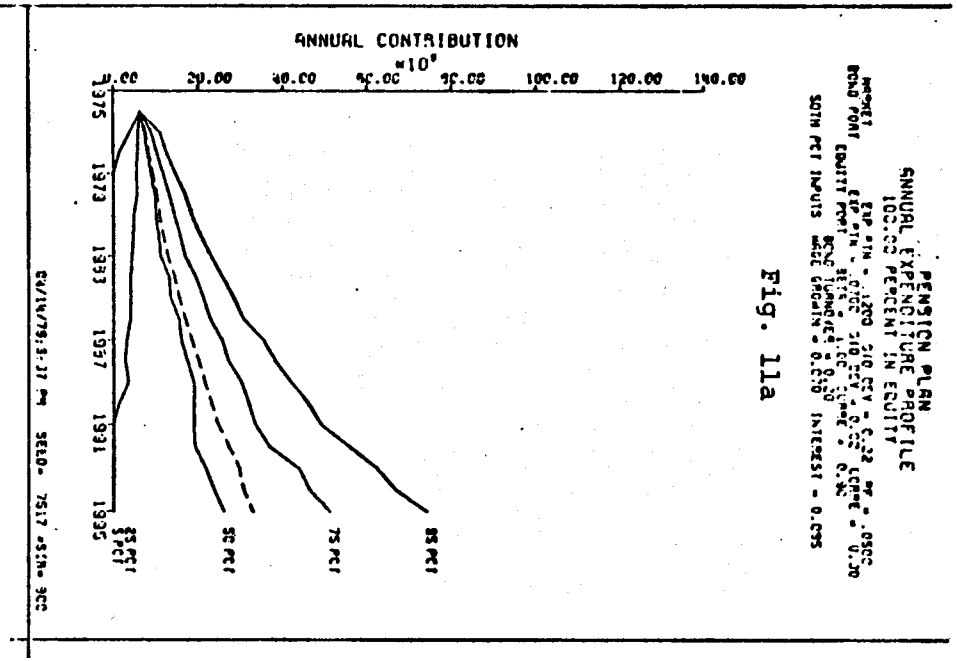
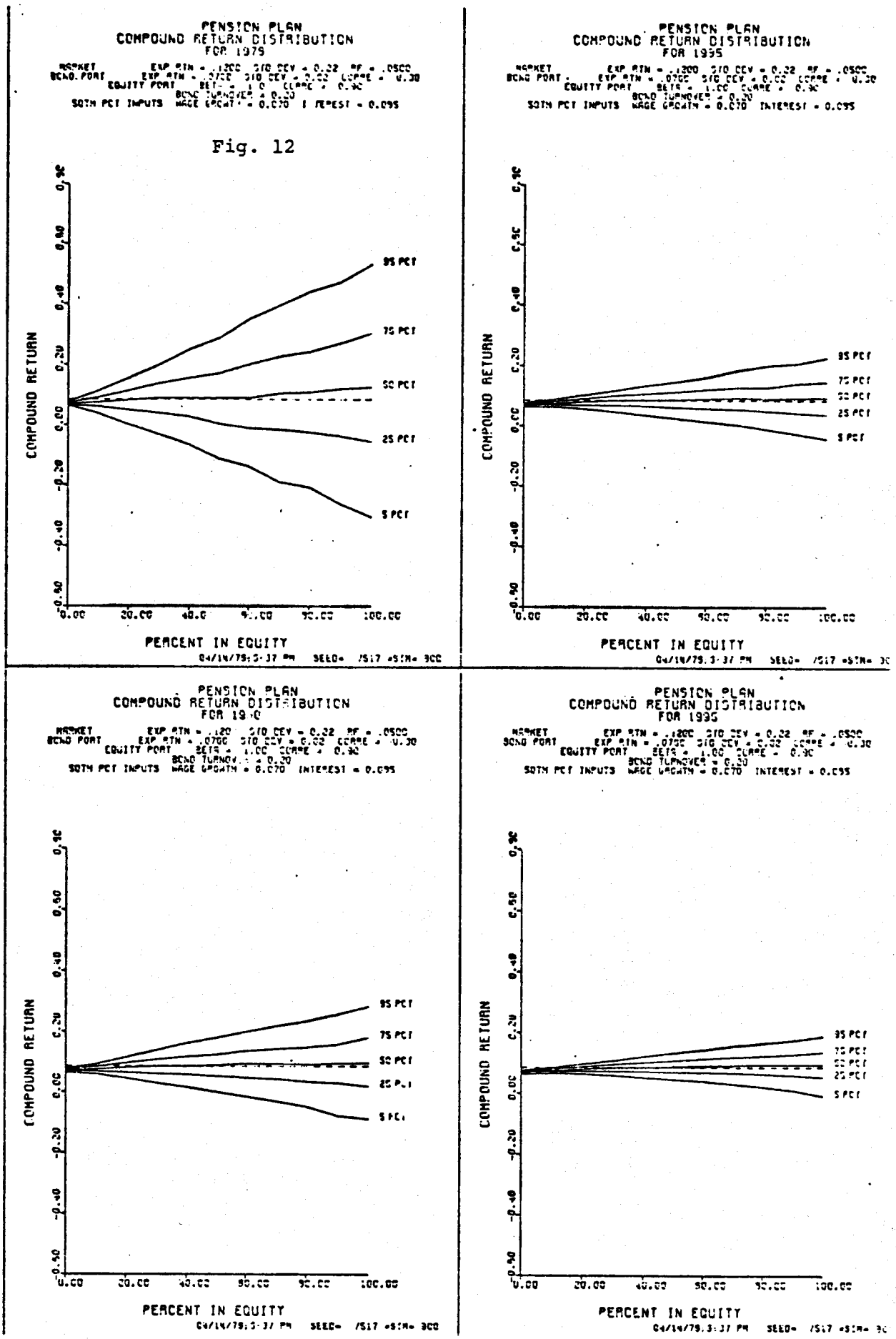


Fig. 11a

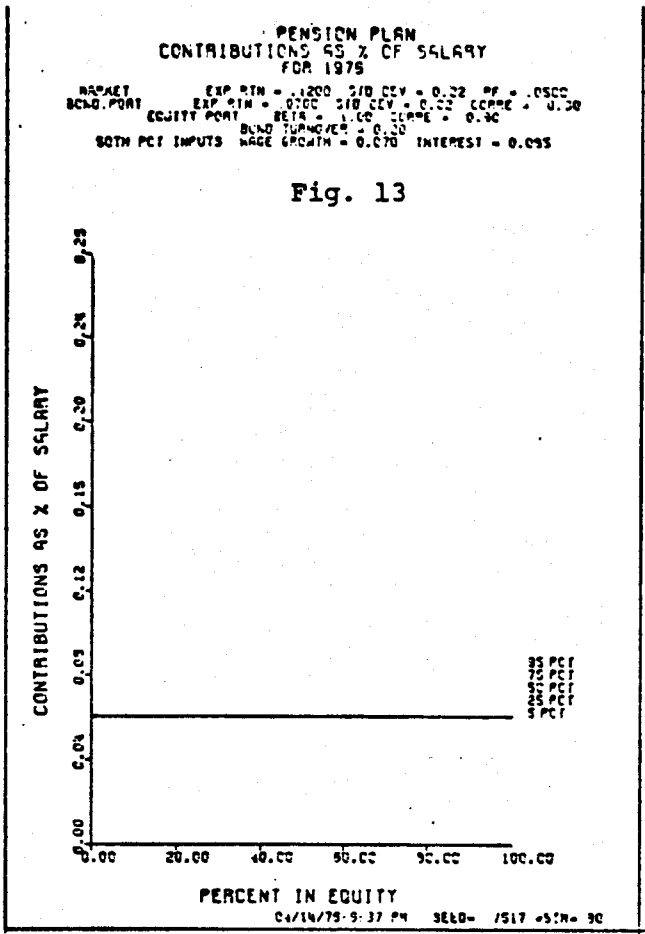
Source: R. Michaud, "Pension Fund Financial Planning", May 1976

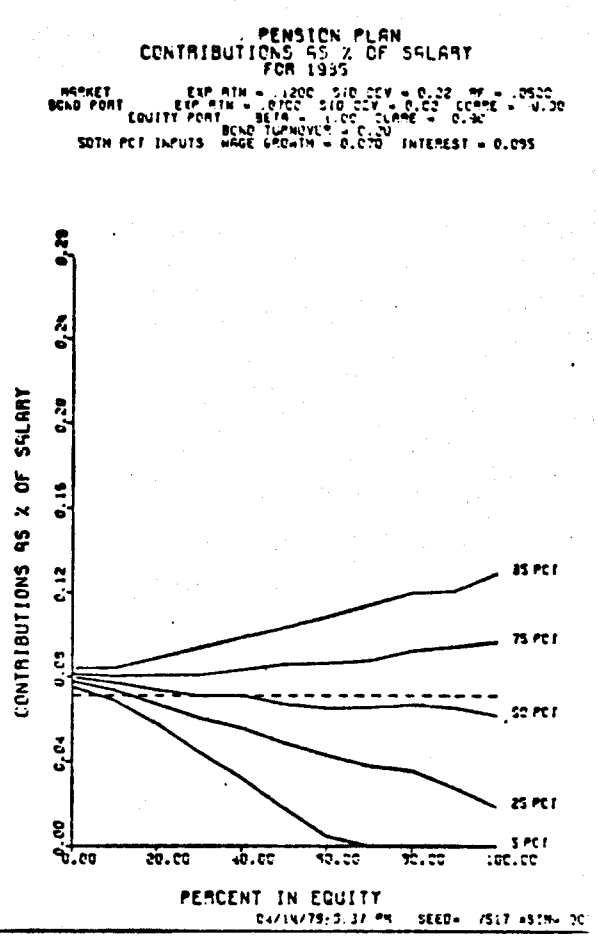
Fig. 12

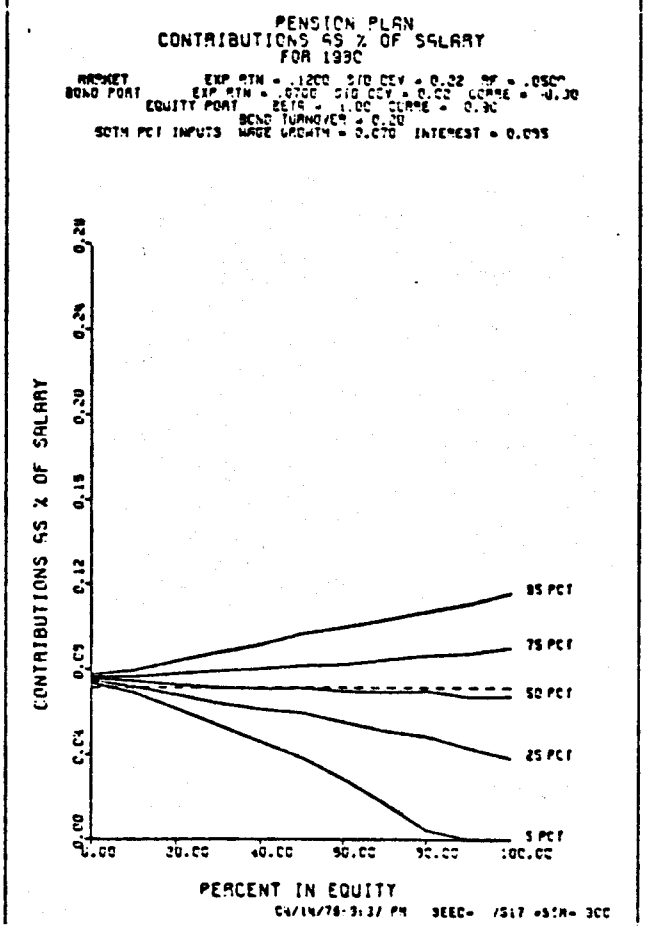


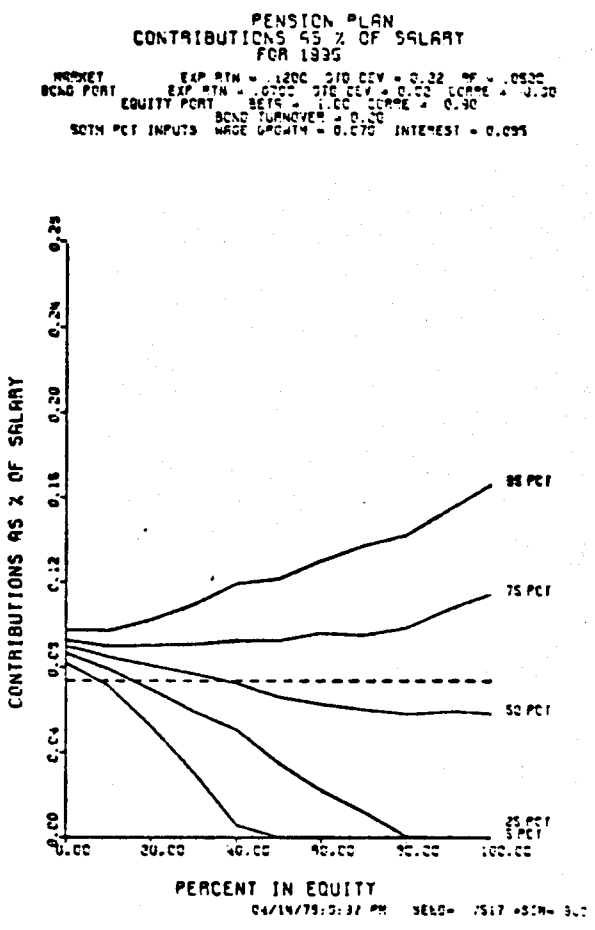
Source: R. Michaud, "Pension Fund Financial Planning", May 1976

Fig. 13









Figures 14-16 examine the effect of a seven percent interest rate assumption on the funding of ABC's pension plan. Median pension liabilities were input to the investment return model. Figure 14 shows that the probability of achieving the actuarial rate and minimizing actuarial losses is significantly greater; at the seventy percent level. More important, the probability increases over time, implying that time is on the side of the pension plan for full funding and lower contributions.

In Figure 15, the status of the funding of the pension plan basically improves, even though pension liabilities are at a higher level.

Figure 16 shows that contributions start at a much higher level, approximately eleven percent of payroll. The trend over time, however, is for the median to decrease to levels comparable to the base case percentages, over a ten to twenty year period.

Although some general improvement in pension fund status is in evidence under the seven percent interest rate assumption over the base case, considering the substantially higher contributions required, it may appear

PENSION PLAN
PROBABILITY THAT COMPOUND RETURN
IS GREATER THAN 0.00%
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.20
BOTH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
PROBABILITY
1.00
0.80
0.60
0.40
0.20
0.00
0.00
20.00
40.00
60.00
80.00
100.00
1985
1980
1990
1979
PERCENT IN EQUITY

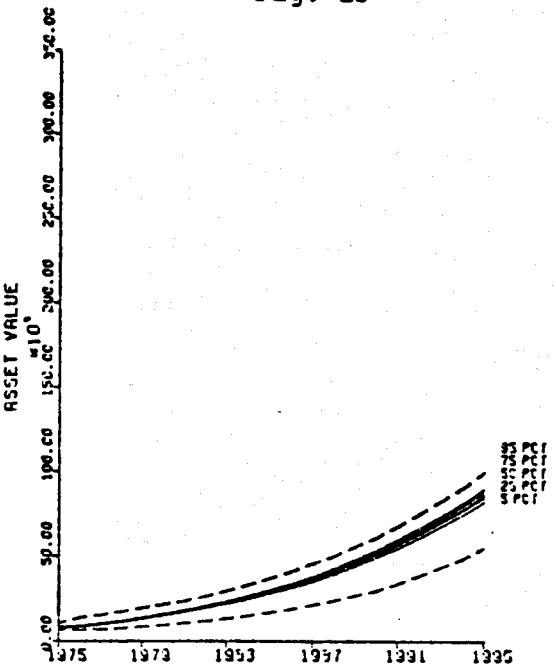
Fig. 14

PENSION PLAN
PROBABILITY THAT COMPOUND RETURN
IS GREATER THAN 5.00%
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.20
BOTH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
PROBABILITY
1.00
0.80
0.60
0.40
0.20
0.00
1985
1980
1990
1979

PENSION PLAN
ASSET VALUE AND PENSION FUND STATUS
0.00 PERCENT IN EQUITY

MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.30
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070

Fig. 15



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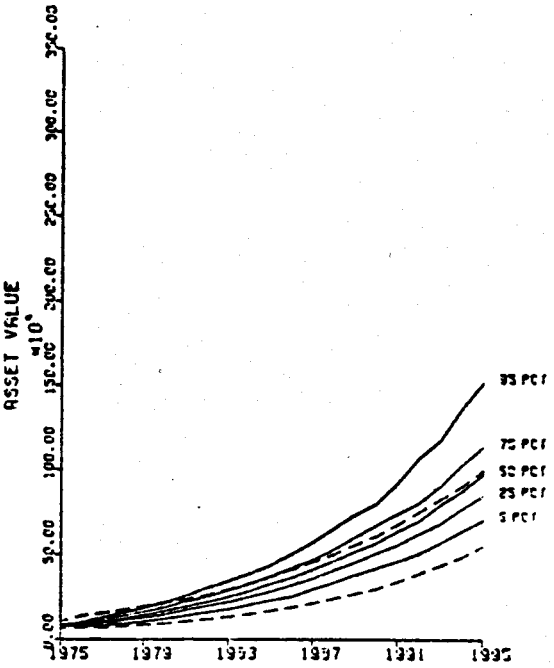
PENSION PLAN
ASSET VALUE AND PENSION FUND STATUS
50.00 PERCENT IN EQUITY

MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.30
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070



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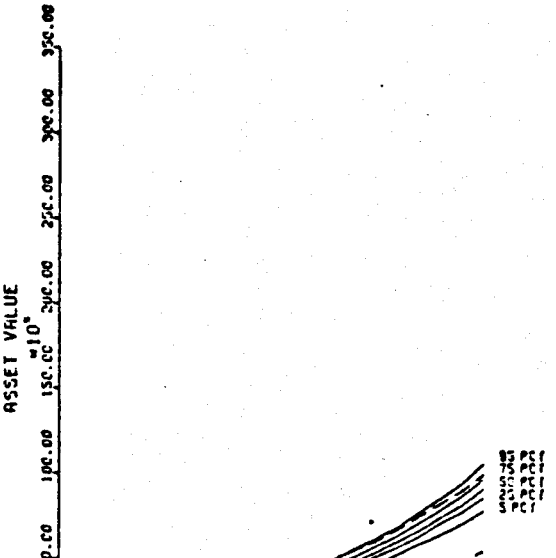
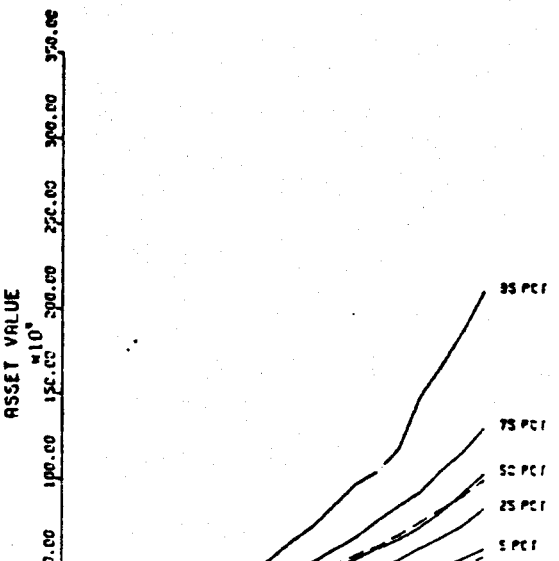
PENSION PLAN
ASSET VALUE AND PENSION FUND STATUS
20.00 PERCENT IN EQUITY

MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.30
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070



PENSION PLAN
ASSET VALUE AND PENSION FUND STATUS
70.00 PERCENT IN EQUITY

MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.02 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.90
BOND TURNOVER = 0.30
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070



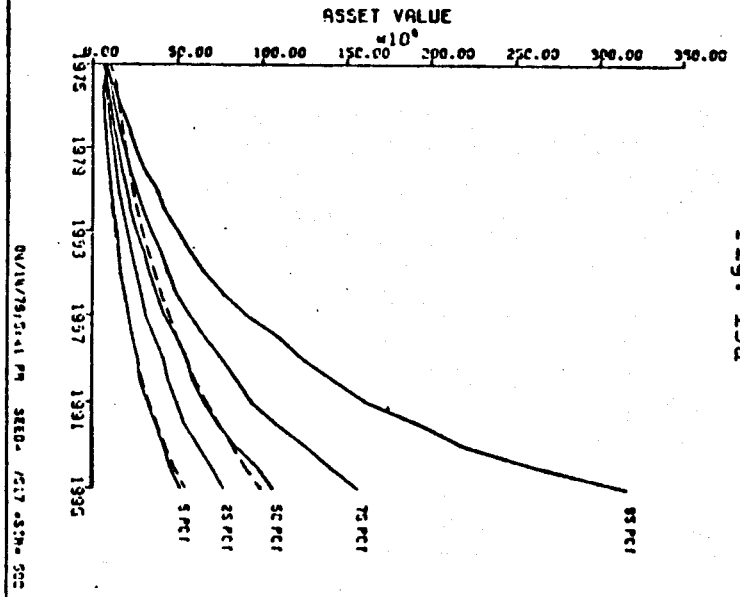

PENSION PLAN
ASSET VALUE AND PENSION FUND STATUS
100.00 PERCENT IN EQUITY
ASSET VALUE
$\times 10^6$
0.00
50.00
100.00
150.00
200.00
250.00
300.00
350.00
1975
1979
1983
1987
1991
1995
5 PCT
25 PCT
50 PCT
75 PCT
95 PCT

Fig. 15a

PENSION PLAN
CONTRIBUTIONS AS % OF SALARY
0.00 PERCENT IN EQUITY
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.03 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.30
BOND TURNOVER = 0.20
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
CONTRIBUTIONS AS % OF SALARY
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
1975
1979
1983
1987
1991
1995
95 PCT
75 PCT
50 PCT
25 PCT
5 PCT

Fig. 16

PENSION PLAN
CONTRIBUTIONS AS % OF SALARY
50.00 PERCENT IN EQUITY
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.03 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.30
BOND TURNOVER = 0.20
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
CONTRIBUTIONS AS % OF SALARY
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
1975
1979
1983
1987
1991
1995
95 PCT
75 PCT
50 PCT
25 PCT
5 PCT

PENSION PLAN
CONTRIBUTIONS AS % OF SALARY
20.00 PERCENT IN EQUITY
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.03 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.30
BOND TURNOVER = 0.20
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
CONTRIBUTIONS AS % OF SALARY
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
1975
1979
1983
1987
1991
1995
95 PCT
75 PCT
50 PCT
25 PCT
5 PCT

PENSION PLAN
CONTRIBUTIONS AS % OF SALARY
70.00 PERCENT IN EQUITY
MARKET EXP RTN = .1200 STD DEV = 0.22 RF = .0500
BOND PORT EXP RTN = .0700 STD DEV = 0.03 CORRE = 0.30
EQUITY PORT BETA = 1.00 CORRE = 0.30
BOND TURNOVER = 0.20
50TH PCT INPUTS WAGE GROWTH = 0.070 INTEREST = 0.070
CONTRIBUTIONS AS % OF SALARY
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
1975
1979
1983
1987
1991
1995
95 PCT
75 PCT
50 PCT
25 PCT
5 PCT

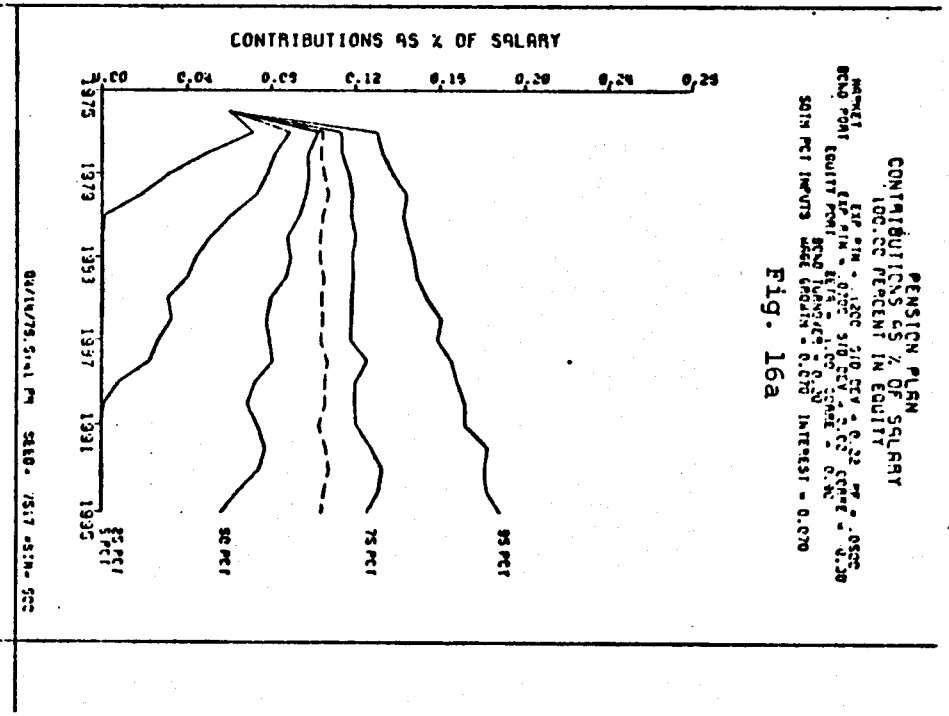

CONTRIBUTIONS AS % OF SALARY
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
1975
1973
1953
1937
1931
1935
95 PCT
90 PCT
75 PCT
55 PCT
PENSION PLAN
CONTRIBUTIONS AS % OF SALARY
100.00 PERCENT IN EQUITY

Fig. 16a

not to be worth the extra cost. Two facts are necessary considerations in this evaluation:

1) the unfunded liability which will be amortized beginning in plan year 1976 is considerably higher with a seven percent interest rate; therefore, plan contributions start at a much higher level for the seven percent interest rate case;

2) pension liabilities typically mature over twenty- thirty- and forty-year periods. This may imply that a twenty year time horizon is not long enough to allow proper evaluation of the effects of actuarial assumptions and investment policy.

The key appears to be the fact that the probability of achieving the actuarial interest rate is higher and increasing with time under the seven percent rate assumption. Using an eight and a half percent interest rate, median pension contributions will remain at approximately the actuarial level. Investment return will typically not assist in reducing contributions. A seven percent interest rate implies a much larger level of contributions at the beginning of the investment horizon. However, investment return will provide actuarial gains with increasingly high probability. Therefore, there will

be an increasing probability that contributions will be less than the actuarial level. This analysis emphasizes the fundamental long-term nature of pension fund financial planning.

9. Summary and Conclusions.

Our analysis has had two basic objectives: 1) to project pension liabilities and cash flow requirements by taking into account the dynamic changes in the workforce and salary-related events which affect the actuarial estimation process; 2) to set investment objectives within the context of the financial obligations of the pension plan and the resources and risk preferences of the plan sponsor.

A computer based model has been developed which projects pension liabilities and asset values, integrates funding policy and records experience gains and losses at each point in time. The model thereby derives the pension contribution profile and the progression of plan funding status over a given planning horizon under a variety of assumptions. By performing simulations for a number of asset mixes, the impact of the investment policy decision can be assessed in order to define an appropriate investment policy.

The ABC pension plan was used to illustrate the pension simulation technique. By performing simulations of the ABC workforce, it was found that it will take approximately twenty years for the workforce to stabilize, during which time there will be systematic ageing. The pension population will grow in size from its present level of eight hundred and thirty to about a thousand in approximately thirty years.

The ageing of the workforce was the major factor responsible for a substantial systematic bias of the pension liabilities from simple roll-forward constant percent of payroll estimates. In contrast to the roll forward estimates of seven percent, by simulating pension liabilities we found that payroll grows at 8% and normal cost grow at 9.1%, on a compound basis, over the twenty-year horizon. The volatility of the pension liability distribution was documented and found to be relatively stable.

Investment return was integrated with pension liability projections in order to forecast pension contributions and funding progress over a twenty-year planning horizon.

Some conclusions derived from our simulations for ABC's pension plan under the base case assumptions were: 1) the probability of actuarial gains was slightly above fifty percent and remained basically constant over time; 2) pension plan status improved slightly with increased equity; a high percent in equity produced a fifty percent chance that the pension plan would be fully funded within twenty years, and a more than five percent chance that assets would be less than vested liability; 3) contributions tend to follow the rising stream of actuarial projections; investment return produces a moderate reduction of median contribution with increasing equity; 4) contributions as a percent of payroll exhibited a fair amount of volatility and remained, on a median basis, approximately level over time.

By comparing simulations from the base case with those using a seven percent interest rate, we observe the following differences: 1) the probability of actuarial gains is at a higher level and increases with time; 2) considerably higher contributions are required, especially in the early years of the planning horizon; 3) plan status was somewhat improved over the twenty-year period; 4) median contributions as a percent of payroll generally decrease over time; with moderate to high equity

portfolios, median contributions as a percent of payroll are reduced to levels similar to the base case over the twenty-year period.

Because of the long maturity of pension liabilities, the beneficial effects of a lower interest rate assumption are not readily apparent, especially in view of the high initial contributions under the seven percent assumption. However, the key result is that, with the seven percent interest rate, the probability of actuarial gains increases with time and therefore time is on the side of the pension plan for full funding and lower contributions.

The results of a financial planning study add a dimension to monitoring and evaluating investment performance. An appropriate measure of investment performance is how investment return enables the pension fund to meet its financial obligations within the overall context of corporate planning and objectives. Using simulation output such as Figures 10 and 11, the track of the status of the plan and pension contributions can be compared to the simulated results in order to determine how well the financial goals of the plan are being met. Orderly funding of the plan within the established goals becomes an investment performance measure of importance.

Apart from various assumptions relating to investment return, there remain many open issues of interest. For this study, asset mix and funding policy remain fixed in any given simulation. It would be of considerable interest to examine alternative funding policies or funding strategies and asset mixes which change over time. The model is capable of examining such issues and many more. As a result, we consider our work as a framework which can be developed and extended to meet the financial planning needs or corporate sponsors.

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