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The Information Ratio of Factor Based Alpha

by

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Abstract

Misconceptions concerning the proper definition of breadth in Grinold's Active Law of Management have suggested that the information ratio of optimized portfolios increases with the number of stocks in the portfolio. We show that when active return depends on factor bets, the IR has an upper bound independent of the number of stocks, but depending on the breadth of the strategy and some maximum information ratio of the joint factor bet.

Grinold's (1989) "Fundamental Law of Active Management" formula states that an optimized portfolio's information ratio is approximately the information correlation (IC) times the square root of breadth; breadth is defined as the number of independent bets. There is evidence that many proponents of the formula have been confused about proper understanding of notion of breadth. Misapplications of the formula are likely due to Grinold and Kahn (1995, p. 130), where they advise using increasingly larger optimization universes for improving the information ratio. As Michaud and Michaud (2005, July, August) show, IC and breadth are not generally independent and the information ratio may or may not increase with increasing breadth.

In "The Information Ratio of Factor Based Alpha", Kraut et al. show that a simple no arbitrage condition forces an upper bound on the information ratio. Moreover, the bound explicitly depends on the square root of the number of factors driving residual return. At best, the information ratio may only be improved marginally by adding more stocks.

The no arbitrage condition relates directly to the idea of alpha. Alpha generally refers to an asset's residual return which isn't correlated to anything else. If a manager could always add more stocks with uncorrelated residual return, he could construct a risk-free strategy with positive return. The no arbitrage assumption states that positive expected residual return results form exposure (a bet) to some risk factor or factors. Considered in light of common practice, this no arbitrage assumption is hardly controversial. Equity portfolio optimizations in institutional practice typically rely on factor models of expected return.

The results show the information ratio for a single or multiple factor valuation model of return is bounded, independent of the number of stocks, by the square root of breadth times a constant. Further, this bound unambiguously depends on the information actually in the strategy. The article does not address the issues associated with valid applications of the fundamental law for asset management or how many stocks a strategy should actually use.

References

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