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Equity Optimization Issues IV: The Fundamental Law of Mismanagement

by

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New Frontier Advisors' Newsletter
July 2005

Abstract

Grinold's (1989) "Law of Active Management" is one of the most widely referenced yet misused formulas in investment theory and practice. The formula describes optimal investment in terms of information level and breadth of investment. It is used as a rationalization for large stock universes and frequent optimizations. While mathematically correct, the formula ignores uncertainty in information and practical constraints in asset management. We show there is no free lunch and that optimized portfolio management should weigh the tradeoff between quality and quantity of information. Errors of practice resulting from using the formula and associated applications may impact as much as two trillion dollars under current management.

The Grinold "Law of Active Management" is one of the most widely referenced and misused formulas in investment theory and practice.[1] The formula describes optimized investment value in terms of information level and breadth of investment. It is used primarily as a guide for mean-variance (MV) optimized equity portfolio design. Grinold and Kahn (1995) (GK) state the implications of the formula: "It takes a modest amount of skill to win (the investment game) as long as that skill is deployed frequently and across a large number of stocks."[2] Their interpretation is appealing but invalid. In contrast, effective optimized active investment strategy involves carefully weighing the *tradeoff* between information level and number of investment bets.

The formula's popularity is due to suggesting techniques for improving MV optimized portfolio performance that any manager can implement. But there are no free lunches in asset management: GK's rules generally lead to mismanagement. Indeed effective strategies are often the opposite of those prescribed by GK. In addition, published extensions of the formula have also given rise to equally ineffective proposals.[3]

The roadmap for the paper is as follows. The first section describes the Grinold formula and its associated MV equity portfolio optimization assumptions. The second section considers the two specific GK prescriptions and shows that they are invalid for optimized portfolio management in practice. Section three describes the implication of estimation error on the unconstrained MV optimization framework used in the formula and introduces simulation studies for reliable optimization strategy valuation. Section four describes theory and recommended optimization practice in the light of estimation error. Section five provides a summary and implications.

1. Grinold's Formula

The formula is based on unconstrained Markowitz (1959) MV optimization.[4] Markowitz optimization is the standard framework for institutional equity portfolio management for nearly fifty years.[5] However, MV optimization is known to have serious investment performance limitations.[6] Resampled Efficiency™ (RE) optimization is a generalization of MV efficiency that is provably effective at improving the investment value of MV optimized portfolios.[7] Unfortunately, the potential investment benefits of a substantially more effective portfolio optimization may often be mitigated by the negative effects of implementing the GK prescriptions.

[1] Grinold (1989).
[2] Ch. 6, p. 130.
[3] Two published papers based on the Grinold formula and affected by similar errors as described here include Grinold (1994) and Clarke, deSilva, and Thorley (2002).
[4] No sign or other constraints on assets are assumed. Even the budget constraint is unnecessary.
[5] In general, institutional applications of Markowitz MV optimization typically assume budget and sign or other constraints on asset values.
[6] Jobson and Korkie (1981).
[7] RE optimization was invented by Richard Michaud and Robert Michaud and is a U.S patented procedure, worldwide patents pending. It was originally described in Michaud (1998, Ch. 6). New Frontier Advisors, LLC (NFA) is exclusive worldwide licensee.

As in Grinold (1989), we take as a given that the information ratio is the standard for measuring the investment value of active equity portfolio investment strategies. Grinold's formula describes the information ratio (IR) of a MV optimized portfolio as the product of the information level (IC) and number of independent sources of information (breadth or BR).[8] Mathematically,

$$IR = IC * \sqrt{BR}$$

where IR = information ratio = (alpha) / (residual or active risk)[9]
IC = information correlation (ex ante, ex post return correlation)
BR = breadth or number of independent sources of information

Grinold's formula is typically used to evaluate the investment value of asset management decisions on optimized portfolio performance. For example, since daily returns are typically more afflicted with noise and less likely to be predictable than longer periods, what is the optimal decision period for a given investment strategy (ignoring trading costs)? Also, if the number of stocks each analyst covers is doubled, will the increased number of investment opportunities outweigh the poorer quality of information on each stock?

Grinold's formula is a statement of ex ante investment value. GK illustrate the basic concepts with a roulette wheel example. In each "play" of the roulette wheel game, the house has a positive IC. The more "plays" of the game, the more value to the casino. The formula gives the (nearly) correct economic value of the casino game. It is strange that the fact that the GK formula applies to casino games has not raised suspicions of its limitations for asset management. Investment decisions are far more complex than well-defined games of chance.

There are two critical assumptions that result in invalid applications of the formula for asset management in practice: 1) Information level and breadth are independent. 2) Unconstrained MV optimization is a useful asset management framework for evaluating optimized portfolio performance. Simulation studies, not the Grinold formula, are the method of choice for understanding optimized portfolio performance.

2. GK Prescriptions

The two GK prescriptions for managing MV optimized portfolios – large stock optimization universes and frequent revisions – are fallacious. Theses errors are the result of a fundamental misunderstanding of information and decisions in investment management.

2.1 Large Stock Universe Fallacy

Will increasing the size of the optimization universe increase the information ratio of a MV optimized portfolio? If the number of stocks is small, a fairly consistent level of IC

[8] The derivation is given in GK, Ch. 6, Technical Appendix. Breadth is also interpretable as volatility.
[9] Residual risk is portfolio risk relative to a benchmark. In many cases residual risk includes systematic as well as active risk. If the strategy is not "beta neutral" residual risk is not pure "active risk." For our purpose here, the distinction is not important and is ignored.

may be maintainable over the optimization universe and each stock valued on its possibly unique characteristics. In this case the estimate of active return (alpha) may be independent of other stocks and the Grinold formula can be written as:[10]

$$IR = IC * \sqrt{N}$$

Now increase the size of the optimization universe. Superficially, the larger the universe the more opportunities to find undervalued stocks all other things the same. But consider the analyst suddenly asked to cover twice as many stocks. Given limitations of time and resources, it is unlikely that the same information level can be brought to bear when analyzing each stock. This is why traditional analysts and portfolio managers tend to specialize in different areas of the market or strategies and limit the number of stocks considered. In general, the IC average is likely to be a decreasing function of the number of stocks in the optimization universe.

Institutional active equity portfolio optimization is often relative to large stock universes such as the S&P500, Russell 1000 or even global stock indices. As a consequence, institutional stock valuation is typically based on factors such as earnings yield. As GK note, if earnings yield is the only factor for stock valuation, there is only one independent source of information and BR = 1 independent of the number of stocks in the optimization universe all other things the same.

In practice, institutional equity managers generally use a multiple valuation framework that may include many factors to form valuation decisions. Synergistically effective multiple valuation requires choosing factors that are reasonably uncorrelated with other factors and simultaneously statistically significantly positively related to ex post return. This is not a simple process.[11] In practice, factors are often chosen from a small number of categories known to be relatively uncorrelated, such as value, momentum, technical, and discounted cash flow.[12] Many popular factors considered for stock valuation are often highly correlated as well as statistically insignificant. Simply adding factors may reduce the average IC level below that of many individual valuation factors. Experience has shown that the breadth of institutional stock valuation models is generally very limited. As a reasonable rule of thumb, the breadth is unlikely to be much greater than five whatever the size of the optimization universe. IC and breadth are not independent of size of the optimization universe and the formula cannot be used to indicate that increasing the size of the optimization universe is likely to improve MV optimized portfolio performance.

2.2 Invest Often Fallacy

In casino games, the more “plays” of roulette the richer the casino. Each play is independent of the others. But are investment decisions independent of prior or subsequent ones?

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[10] This assumption implies a diagonal covariance matrix.
[11] See Michaud (1990) for further discussion of these issues.
[12] A value factor such as earnings yield may have little correlation with technical or momentum factors.

Consider two investment management scenarios. The first scenario is buy and hold for the year. In the other scenario the manager considers subdividing the year into increasingly smaller decision time periods: quarterly, monthly, weekly, daily, hourly or less. To be consistent with the GK assumptions, each decision in the subdivided periods must be independent of the others. As the decision periods shrink in length, the less information or IC available. All that can be said with reliability is that the decision periods should be sufficiently frequent to be able to freely use relatively independent reliable information in the investment process.[13] The appropriate period depends on the nature of the strategy rather than any simple rule for when to optimize a portfolio.[14] The Grinold formula cannot be used to indicate that increasing the frequency of decisions is likely to improve MV optimized portfolio performance.

3. Unconstrained MV Optimization and Estimation Error

The Grinold formula assumes unconstrained MV optimization. One impractical implication of this assumption is that an investor is essentially able to borrow without limit to make bets on each and every opportunity discovered. In reality, investing more in one asset typically means investing less in another. This assumption is one of the keys for understanding the paradox why the formula is useful for casino games but unreliable for investment management.

The critical limitation afflicting the reliability of the Grinold formula prescriptions is that it completely ignores the impact of estimation error on MV optimized portfolios. Unlike casino games, there is much estimation error in MV optimized portfolios. Including estimation error in MV optimized portfolio performance generally invalidates and often reverses the GK prescriptions.

3.1 Simulation Studies for Unconstrained MV Optimization

In their classic simulation studies, Jobson and Korkie (1981) investigated the investment value of unconstrained MV optimized portfolios with estimation error. They found that, on average, the Sharpe ratios of the unconstrained MV optimized portfolios were remarkably poor estimates of their true value. Astonishingly, they find that equal weighted portfolios were far more truly efficient than unconstrained MV optimized portfolios on average.

It should be noted that the Jobson and Korkie studies actually represent “best cases” for MV optimization. The simulations assume a stationary return distribution and estimates that are never based on perverse assumptions, conditions not met in practice. The results are interpretable as showing that for estimation error inherent in financial data, unconstrained MV optimized portfolios have essentially no investment value. They conclude that investors should avoid unconstrained MV optimization. The Grinold formula inherits the limitations of evaluating optimized portfolio value implicit in the unconstrained MV optimization framework.

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[13] Some special cases may include proprietary trading desk strategies where the information level is maintained at a reasonable level and trading costs are minimal.

[14] For example, a value manager will probably have longer periods between optimizations than a growth stock manager. Trading costs are another important consideration.

Increasing the number of securities in a Jobson and Korkie study increases estimation error and consequently reduces likely MV optimized portfolio performance, all other things the same.[15] Simulation studies, not the Grinold formula, are the reliable frameworks for understanding the investment value of MV optimized portfolio strategies.

3.2 Estimation Error and Constrained MV Optimization

Allowing negative portfolio weights does not change the MV optimization algorithm, but does fundamentally change the risk associated with investment. A sign-unconstrained portfolio exposes the investor to unlimited liability risk. Such additional risk is typically not in the risk model used in a portfolio optimization. It is for this reason that regulations governing institutional portfolios often require sign constraints.

Do sign constraints limit the performance of active equity managers when estimation error is included? Are unconstrained MV optimized portfolios better in practice? Frost and Savarino (1988) show that sign constraints improved the value of MV optimized portfolios relative to unconstrained optimization when estimation error is included. Financially meaningful constraints, may improve, not limit, the investment value of MV optimized portfolios, a prescription in conflict with many papers associated with the GK formula.[16]

3.3 Improved MV Optimization

Resampled Efficiency™ (RE) optimization is designed to address estimation error for MV optimized portfolios. RE optimization properly uses the resampling technique to improve optimized portfolio value by including the uncertainty endemic in all investment information in the optimization process. Simulation studies in Michaud (1998, Ch. 6) showed that RE optimized portfolios have superior investment value.[17]

Simulation studies are a powerful new tool that revolutionizes the ability to understand and improve MV portfolio optimization in practice. Many results are now available and often inconsistent with conventional MV optimization wisdom.[18]

4. Theory and Practice

The presence of estimation error reflects the investment reality that stock valuation information is often not statistically significant. Typically useful information is available for a relatively small number of stocks in practice. Merton (1987) shows that the appropriate optimization universe should be defined on securities with useful information. You should optimize only on what you know.

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[15] In institutional equity portfolio optimization, adding stocks can reduce tracking error risk all other things the same. But this is a special case associated with institutional practice that does not invalidate the result.
[16] For example Clarke, deSilva, and Thorley (2002).
[17] More recent simulation studies by Markowitz and Usmen (2003) indicate that RE optimization is superior to MV optimization even with inferior risk-return estimates.
[18] We will be reporting on some of these results in future reports.

In practice, the Merton prescription has an important limitation. Equity portfolio managers are generally asked to outperform an index while holding the tracking error risk within a specified range. However, defining the optimization universe solely for statistically significant alpha stocks is likely to expose the portfolio to more index tracking error risk than may be acceptable. Michaud and Michaud (2005) provide a resolution. They propose adding a composite asset of index weights of all non-statistically significant alpha securities in the index to the significant alpha stock optimization universe. Tests show that the composite asset technique provides a convenient framework for MV optimization consistent with Merton's theoretical framework while satisfying the need for controlling tracking error risk.

5. Summary and Implications

If you are a casino manager then the Grinold formula will take good care of you. However, if you are an asset manager, you should avoid using the formula for designing your optimized portfolios. Effective active asset management includes understanding the tradeoff between information and the number of investment bets. The critical limitations of the formula include the non-independence of information and breadth and the irrelevance of an unconstrained MV optimization evaluation framework. Simulation studies provide the proper framework for evaluating optimized portfolio investment strategy value. Simulation results often directly contradict GK prescriptions and other applications of the Grinold formula. Indeed, any paper on MV optimization strategies that does not consider estimation error is likely to be out-of-date and unreliable.

The limitations of the GK formula need to be better known in order to avoid self-defeating procedures and enable superior optimization methods to add investment value. Many institutional asset managers under the influence of the GK formula are using optimized portfolio designs that are likely to diminish investment value. Instead, use simulation studies for reliable analysis of optimized portfolio investment strategies. Analyze the valuation process for statistically significant estimates. Use RE optimization for the stocks you know and our composite index procedure to manage tracking error risk if necessary.

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