Equity Optimization Issues II:
Large Stock Universes and Scaling Alphas

by

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Abstract

In order to obtain the provable benefits of Resampled Efficiency, a number of common and ad hoc equity portfolio optimization techniques need to be avoided or corrected. This article focuses on two: the use of large stock universes and incorrect alpha scaling.

This essay is the second in a series begun in November 2004 on avoiding self-defeating practices and frameworks that limit the potential investment value of effective equity portfolio optimization.[1]

Equity Optimization Practices

Equity portfolio optimization is often a specialty activity for an analytically sophisticated analyst in an asset management firm. Optimized portfolios are typically not used out-of-the-box but go through a review process by senior investment professionals that can be extensive and time consuming. Investment organizations often impose many ad hoc constraints and procedures and re-optimize a number of times before submitting an optimized portfolio to a trading desk.

Ad hoc procedures typically function as cosmetic controls for acceptability and marketability of optimized portfolios. Because classical mean-variance (MV) optimization has little, if any, inherent investment value, these procedures often have neither positive nor negative investment consequences. However, in the context of an optimizer that is estimation error sensitive, these same ad hoc procedures may often have self-defeating consequences. In order to obtain the provable benefits of Resampled Efficiency™ (RE) optimization, a number of common ad hoc practices need to be avoided or corrected.[2] This newsletter focuses on two of the most serious: the use of large stock universes and incorrect alpha scaling.[3]

Resampling and Portfolio Optimization

Under very general conditions, Markowitz MV portfolio optimization tells us the correct way to invest given our estimates of risk and return. But risk-return estimates are always uncertain in practice. Estimate uncertainty is totally ignored and is the cause of the poor investment performance that characterizes MV optimization.

Estimate uncertainty implies that there are many equivalently optimal MV efficient frontiers. Resampling is a Monte Carlo procedure that is the method of choice for understanding uncertainty in risk-return estimates. RE optimization is a generalization of MV optimization that includes uncertainty in the definition of portfolio optimality. RE optimality has many intuitively attractive investment properties. It is a necessary condition for investment effective MV optimal portfolios. Importantly, RE optimization is the only provably effective portfolio optimization procedure in the world today.

Our purpose in this note is to provide guidance on avoiding ineffective or self-defeating practices and provide recommendations for more effective equity portfolio optimization.

[1] Richard Michaud and Robert Michaud, 2004 Newsletter, New Frontier Advisors, LLC (NFA), November.

[2] Richard Michaud and Robert Michaud are co-inventors: U.S. Patent awarded December 1999, worldwide patents pending. New Frontier Advisors, LLC is the exclusive worldwide licensee.

[3] Alpha inputs into an equity optimization are formally systematic risk-adjusted forecast returns. However, there are many variations used in practice. Intuitively, alpha represents the forecast return associated with an active investment strategy risk-adjusted.

I. Large Stock Universes

Many investment organizations develop optimized portfolios from a large stock universe. The optimization lists may include 3000 or 7000 stocks for domestic managers and 30,000 or more for international managers. Compatibility with a large benchmark and seeking superior portfolio performance are two frequent reasons for including a large number of assets in an optimization

Large Universe Rationale
A simple rationale for large stock universe optimization is that, as the number of stocks in the investment universe increases, predicted optimized portfolio return for given risk generally increases. But MV optimization is insensitive to estimation error. Predicted optimized return is an upward biased estimate of actual return, and the bias increases with increasing size of the universe all other things the same. Increasing stock universe size may often decrease actual observed return on average.

Grinold's (1989) "Law of Active Management" is sometimes used, incorrectly, to rationalize large stock universe optimization. Roughly summarized, the law states that the value of an investment strategy increases as the available number of independent sources of information increases. But more stocks do not imply more information sources for the optimization. Suppose a P/E ratio strategy applied to either a 100 or 10,000 stock universe. In either case, there is only one independent source of information. In practice, many multiple valuation investment strategies involve highly correlated factors and generally reflect a relatively small number of truly independent sources of information.[4] Holding the number of sources of information constant, investment performance often decreases as the investment universe increases, as the greater unreliability dominates the increased opportunity of the larger universe. Many top traditional active managers limit their "buy" list to a hundred or fewer of well researched stocks whatever the index stock universe size.

Finally, active equity portfolio management is generally associated with investment relative to some index stock universe. Consequently all stocks in the index may be assumed investable and the size of the universe is immaterial. However, since there is often little statistically significant information for many assets, the portfolio may be best served by benchmark weighting or otherwise limiting investment in stocks with unreliable information and actively investing only in stocks with reliable information.

Selecting "Optimal" Assets
The purpose of a portfolio optimizer is to optimally allocate wealth among risky assets. The optimizer is insensitive to whether a stock is investable or not. The burden of determining investability is on the analyst defining the optimization universe not on the optimizer.

[4] A discussion of such issues is given in Michaud (1990).

A classical optimizer will often ignore many assets in an asset universe and appear to select "optimal" ones. In contrast a RE optimizer will include many assets in the investment universe at some point on the RE frontier. Is the classical optimizer useful in selecting stocks to invest? No. A classical optimizer assumes that a stock with 5% alpha is more attractive than one with 4.95% alpha and the latter stock might be completely ignored in a classical optimization. But this "selection" process is empty of investment content and simply an artifact of the assumption of absolute certainty implicit in MV optimization.

No Information Level

Increasing the size of the optimization universe decreases the level of information on average for each stock and increases the estimation error for the optimization all other things the same. The estimated residual risks of many small capitalization stocks can be as high as 100%. It is also likely that the factor exposures (betas) for many of the stocks are unreliable. A small alpha coupled with a large residual risk has enormous estimation error. The likely true alpha values may range from large positive to large negative. In practical terms, virtually no reliability can often be assigned to a stock alpha with large residual risk. In this context no optimizer can be relied upon to produce useful investment portfolios.

Incomplete Information

Financial theory does not support the use of large stock universes in an optimization. Merton (1987) develops a framework for equilibrium in the presence of incomplete information. His results show that the appropriate optimization universe excludes stocks with no special investment information. In practice stocks with insignificant alpha relative to residual risk are likely candidates for exclusion. It is also useful to note that Merton's theoretical results are consistent with much traditional investment wisdom of finding a relatively small number of "good investments." Analytically sophisticated managers and traditional investors are advised to do the same thing: buy what you know.

Context Dependent Optimal Portfolios

A statistical view provides a very useful perspective on understanding the limitations of large stock universe optimization. When unconstrained, MV optimization is equivalent to least squares linear regression. MV optimization asset allocations are linear regression coefficients.

Linear regression is well-known to be context dependent. Removing a factor or adding another may often change the regression coefficient of the original factors. In the same way, adding stocks or removing them may often change the optimal allocations to an asset. Since the context drives the optimization, it is important to define the universe in an investment appropriate way. Including an additional inappropriate stock in the optimization universe may result in a decreased allocation to a good stock or an increased allocation to a bad stock.

Why Asset Allocation is Different

In a typical asset allocation optimization, the number of assets is usually small and each has been vetted for investability and investment attractiveness. In this context an RE optimization is likely to find investment intuitive allocations without the need for any ad hoc structure. The major difference in an equity portfolio optimization is that in much practice the investment universe isn't confined to investable and investment attractive investments.

Best Practices
Large stock universes for equity portfolio optimization have been recommended by many well-meaning and otherwise knowledgeable academics and investment practitioners in published academic and practitioner journals and textbooks.[5] Unfortunately, in this and many other cases, procedures which may do no investment harm and some cosmetic good for classical MV optimization are self-defeating in the context of an optimization procedure that can actually add investment value. We recommend that optimization investment universes always be confined to "what you know" investments.

II. Proper Scaling of Forecast Returns

It is widely understood that return estimates have to be properly scaled if an optimizer is to prove investment effective. This is true in the usual case when trading costs are included and when a utility function framework is used.[6]

Scaling Forecasts
Asset managers use many different frameworks for forecasting equity returns or alphas. These include ranking methods, discounted cash flows, econometric modeling, and multiple valuation methods. The fundamental principal is that scaled forecasts reflect the "return on average" associated with the forecast. A correct procedure for proper scaling for return rankings is given in Farrell (1983). The more general case where forecast return has a scale similar to actual return was given in Michaud (1989, Appendix).[7] The appropriate scaling procedure is the regression coefficient of the forecasts with respect to actual return.[8] In order to enhance predictability, Stein linear regression estimation may be used.[9]

Scaling in Practice
Scaling is generally associated with the "IC" or information correlation, a number less than one, that represents the level of information (measured or assumed) in the forecasts.[10] All other things the same, the IC adjustment, which is part of the linear regression coefficient formula, shrinks forecast return volatility and is often used to justify shrinking the size of alphas.[11] Michaud (1989) warned that the proper IC adjustment does not uniformly shrink

[5] We'd prefer not to provide references.
[6] We argued in Michaud and Michaud (November 2004) that explicitly optimizing an expected utility function has important pitfalls and is not recommended.
[7] More specifically, forecast return is on a ratio scale.
[8] Farrell (1983) is a specific case of the Michaud (1989, Appendix) procedure.
[9] Stein methods are discussed in Michaud (1998, Ch. 8).
[10] The IC is sometimes referred to as the information coefficient.
[11] One influential article rationalizing shrinking alpha with the IC in the general case is Grinold (1994).

forecast alpha. Proper scaling must consider the interaction of the level of information (IC) with assumed volatility. High volatility securities such as growth stocks often have a low IC and lower volatility securities such as value stocks may have a higher IC. In relatively homogeneous capitalization stock universes, IC times volatility may not vary much. The "shrinking alpha" fallacy comes from a fairly widespread error in the investment community that the IC can be considered a constant while volatility may vary.[12] The correct intuition for "alpha shrinking" comes from Stein estimation methods.

Insignificant Alphas

In practice, optimization alpha forecasts often reflect a remarkably narrow range of values even for large stock universes, from say -3% to 3%. The obvious objective is to control the behavior of traditional MV optimization in the context of trading costs. Many stocks will not pass the trading cost hurdle and MV optimization will recommend relatively moderate sized cosmetically acceptable portfolios even for large stock universes. But properly understood, such a procedure is optimization in white noise. Many commercial risk models typically estimate stock residual risk at 30% or 40 % and often as high as 100%.[13] The alphas in a -3% to 3% range very generally have no investment or statistical significance. The investor can only hope that the constraints imposed on the optimization have some investment value.

Proper scaling of forecast alphas in equity portfolio optimization is subject to major pitfalls for many in the investment community today. An RE optimizer with properly statistically estimated inputs will often result in quite reasonable and acceptable optimized portfolios. The potential for significantly improved optimized portfolio investment performance is available for those able to avoid the errors of the past.

Conclusions

Many procedures associated with MV equity portfolio optimization in practice are ad hoc and often fallacious. Their primary benefit is cosmetic; the portfolios are made to conform to preconceived notions of investment acceptability. Cosmetic procedures generally do no additional harm to an optimizer that is insensitive to estimation error. The problem arises when these same cosmetic procedures are imposed on RE optimization, which uses investment information appropriately. Avoiding these practices with an RE optimizer is essential for realizing the investment potential of estimation error sensitive optimization.

In an ideal world, equity portfolio optimization should be a straightforward process focused on good estimates of risk and return properly scaled. This ideal world is impossible with traditional MV equity portfolio optimization technology but fairly close at hand with RE optimization. With proper inputs, RE optimization may often avoid the

[12] Grinold (1994) ignores the interaction of IC with volatility.
[13] A cutoff value used by many commercial risk models.

investment unintuitive and often self-defeating character of current practice with the potential of substantially improving performance.

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